Advances in Fast MPC & Constrained MRAC for Aerial Robotics

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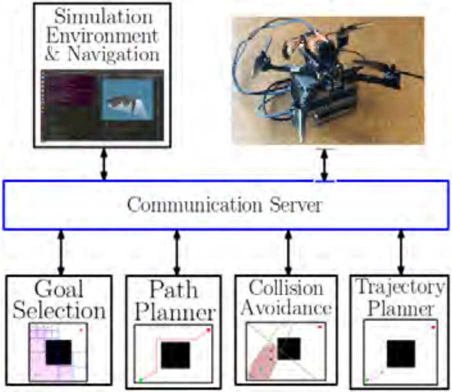
Advanced Control Systems Lab

Project 1: Tactical Guidance

- Objective 1: Create guidance system for autonomous UAVs in *potentially* hostile environments;
    - Perform goal-to-set missions
    - Perform mapping (area coverage) missions
- Objective 2: Create taxonomy of flight behaviors based on *perceived* risk;
- Bio-Inspired Approach: Mimic the behavior of prey animals seeking food in unknown environment;

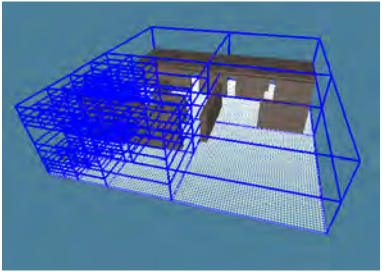
Architecture Overlook

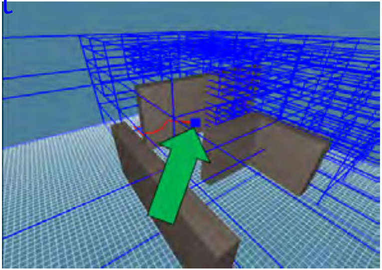
- Goal selection algorithm based on Octree structure;
- Path planner exploits the environment for stealth;
- Trajectory planning employs output feedback linearization and fast MPC;
- Collision-avoidance based on semidefinite programming;
- All modules execute in parallel.



Algorithm to Select Goal Points

- Partition the voxel map
    - Explored, free vox.;
    - Explored, occupied vox.;
    - Unexplored (free) vox..
- Determine sufficiently unexplored partitions;
- Determine the largest & closest partitions;
- Goal point that interpolates their centers;
    - Systematic exploration: closest partition
    - Greedy exploration: largest partition





Tactical Path Planning

Path planning problem solution over a voxel map

$$\min f_k \triangleq g_k + h_k,$$

where

$$g_k \triangleq \sum_{q=1}^{k} \left[\kappa(\mathrm{d}_2(\hat{r}_q, \mathcal{O}))\mathrm{d}_2(\hat{r}_q, \hat{r}_{q-1})\right],$$

$$h_k \triangleq (1 - \mu_2)\mathrm{d}_2(\hat{r}_k, \mathcal{G}),$$

$$\kappa(\alpha) \triangleq 1 - \mu_2 e^{4\mu_1\mu_3 - \left[\mu_3\alpha + \mu_1\alpha^{-1}\right]^2}$$

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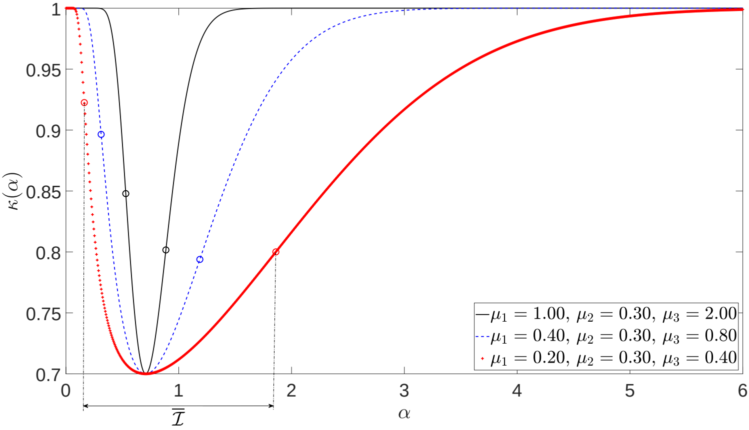
Tactical Path Planning – Search Bias



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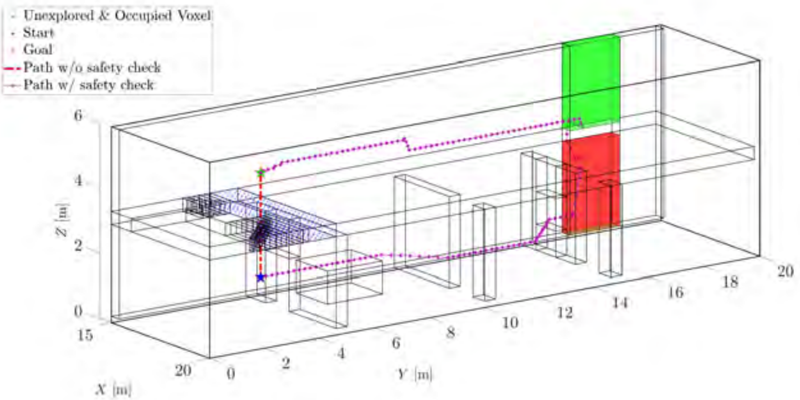
Algorithm to Ensure Path Planner Safety

Avoid areas not detectable by the UAV's field of view

- Too close to the yaw axis



Fast Collision Avoidance

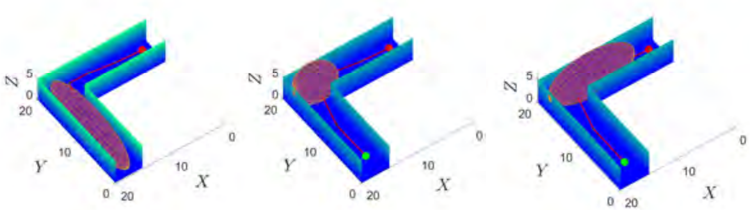
1. Employ semi-definite programming (SDP) to find ellipsoid

$$\left\{w \in \mathbb{R}^3 : (w - r_k(i\Delta T))^{\mathrm{T}} P_k(i\Delta T)(w - r_k(i\Delta T)) + c_k(i\Delta T) \leq 0\right\},$$

containing the UAV & tangent to 1 occupied voxel

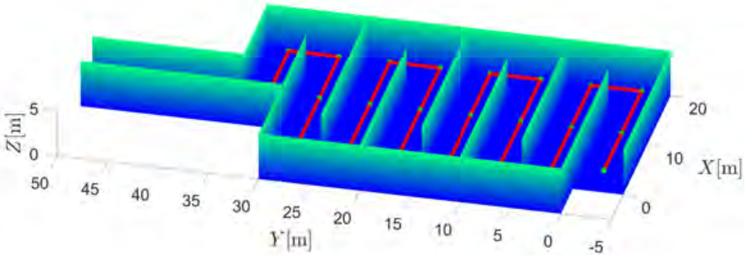
- Discrimination problem

2. Discretize this ellipsoid



Comparison with IRIS & SFC

50 simulations & 24 waypoints in S-shaped hallway



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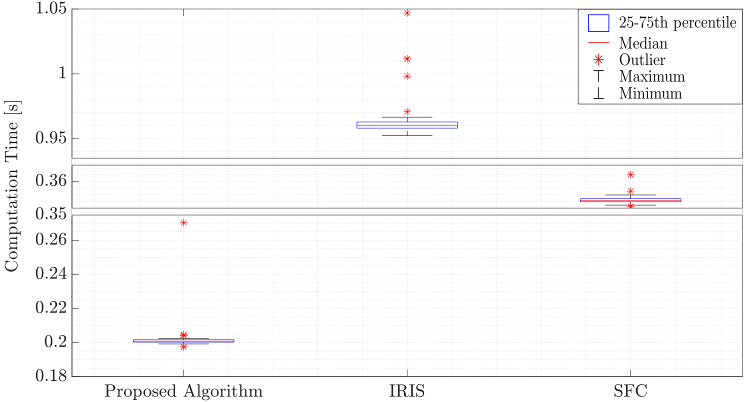
Comparison with IRIS & SFC

50 SIL simulations & 24 waypoints in S-shaped hallway



Tactical Trajectory Planning I

Cost function:

$$\tilde{J}[\hat{r}_k, u_k(\cdot)] \triangleq \ell_{\mathrm{f}}(\tilde{r}_k(n_{\mathrm{t}}\Delta T)) + \sum_{i=0}^{n_{\mathrm{t}}-1} \tilde{\ell}(r_k(i\Delta T), u_k(i\Delta T)),$$

where

$$\tilde{\ell}(\tilde{r}_k, u_k) \triangleq \begin{bmatrix} \tilde{r}_k \\ u_k \end{bmatrix}^{\mathrm{T}} \tilde{R} \begin{bmatrix} \tilde{r}_k \\ u_k \end{bmatrix} + \tilde{q}_r^{\mathrm{T}} \tilde{r}_k + \tilde{q}_u^{\mathrm{T}} u_k,$$

$$\ell_{\mathrm{f}}(r_k) \triangleq (r_k - \hat{r}_{k+1})^{\mathrm{T}} R_{r,\mathrm{f}} (r_k - \hat{r}_{k+1}) + q_{r,\mathrm{f}}^{\mathrm{T}} (r_k - \hat{r}_{k+1}),$$

$$\tilde{r}_k(i\Delta T) \triangleq \mu_4 [r_k(i\Delta T) - \hat{r}_{k+1}] + (1-\mu_4) f_{\mathrm{sat}}(\mu_5(\hat{r}_k - r_{\mathcal{O}})) [r_k(i\Delta T) - r_{\mathcal{O}}],$$

$$f_{\mathrm{sat}}(w) \triangleq \mathrm{sat}(\|w\|)/\|w\|$$

Tactical Trajectory Planning II

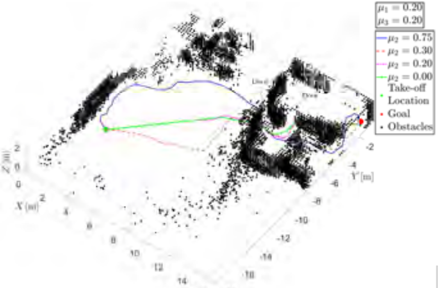
Dynamic Constraints:

$$x_k((j+1)\Delta T) = A x_k(j\Delta T) + B u_k(j\Delta T),$$

$$\begin{bmatrix} r_k(i\Delta T) \\ v_k(i\Delta T) \end{bmatrix} = \begin{bmatrix} r_{\rm init} - r_{\rm e} \\ v_{\rm init} \end{bmatrix}, \qquad \begin{bmatrix} r_k(n_{\rm t}\Delta T) \\ v_k(n_{\rm t}\Delta T) \end{bmatrix} = \begin{bmatrix} \hat{r}_{k+1} - r_{\rm e} \\ v_{\rm end} \end{bmatrix},$$

Collision avoidance, saturation & pointing constraints:

$$F_k(i\Delta T) \begin{bmatrix} x_k(j\Delta T) \\ u(j\Delta T) \end{bmatrix} \leq\leq f_k(i\Delta T),$$



Fast Trajectory Planning

Reduce MPC problem to QP with block-tridiagonal matrices

$$\min I_{i,\mathrm{lb}}(z_{i,k}) \triangleq z_{i,k}^{\mathrm{T}} H_{i,k} z_{i,k} + g_{i,k}^{\mathrm{T}} z_{i,k} + \nu_1 f_{\mathrm{lb}}(z_{i,k}) + \sum_{q=1}^{(l+10)(n_{\mathrm{t}}-i)} \frac{1}{\nu_{4,i,k,q}} \log\left(1 + e^{\nu_{4,i,k,q}\left[p_{i,k,q} z_i - \hat{h}_{i,k,q}\right]}\right)$$

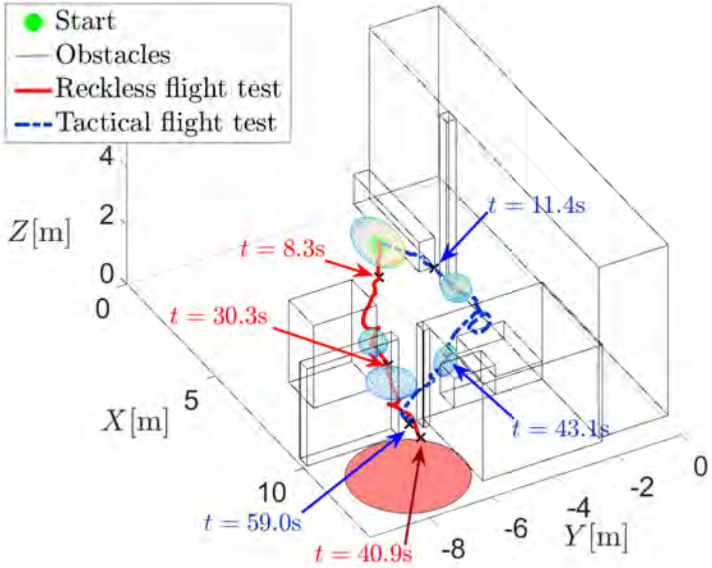
$$\mathrm{s.t.}\, C_i z_{i,k} = b_{i,k},$$

where hard inequality constraints are captured by

$$f_{\mathrm{lb}}(z_{i,k}) \triangleq -\sum_{q=1}^{(l+10)(n_{\mathrm{t}}-i)} \log\left(h_{i,k,q} - p_{i,k,q} z_{i,k}\right)$$

- Soft constraints: Kreisselmeier-Steinhauser function avoids buffers & keep banded structure

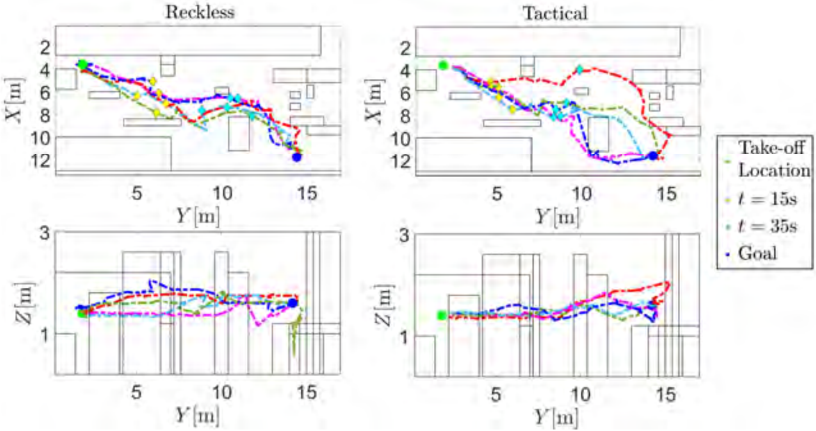
Flight Tests Results – Overview



Selected Flight Test Results I

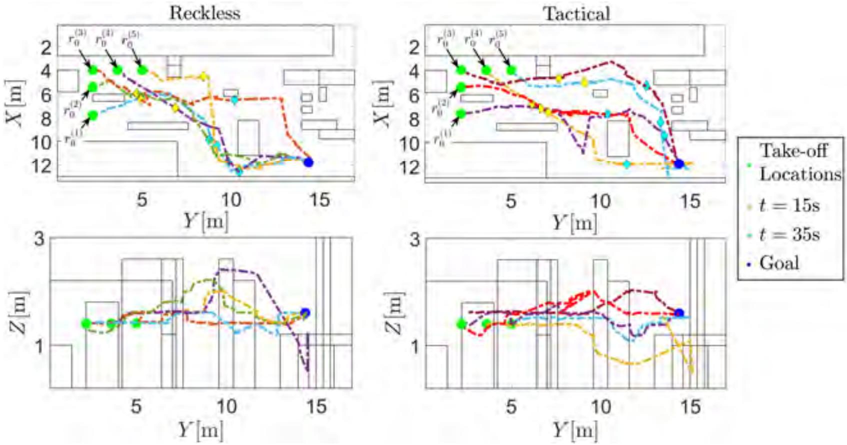
5 tactical & 5 reckless flight tests

- Tactical flights coast obstacles more;
- Tactical flights are less predictable;

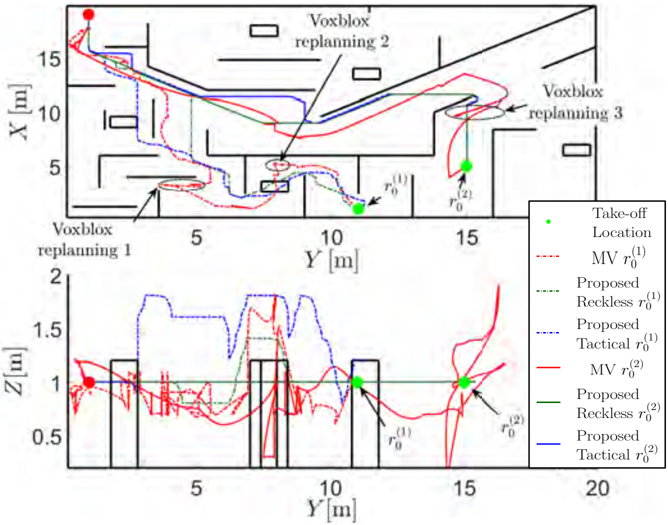


Selected Flight Test Results II

Tactical & reckless flight tests with 5 initial conditions



Comparison with Voxblox Planner (ETH)



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Project 2: Prescribed Performance MRAC

Quick overview of *model reference adaptive control*:
Find a control law for the plant

$$\dot{x}(t) = Ax(t) + B\Lambda\left[u(t) + \Theta^{\mathrm{T}}\Phi(t, x(t))\right],$$

so that $\lim_{t\to\infty}\|x(t) - x_{\mathrm{ref}}(t)\| = 0$, where $A \in \mathbb{R}^{n\times n}$, $\Lambda \in \mathbb{R}^{m\times m}$ & $\Theta \in \mathbb{R}^{N\times m}$ are unknown &

$$\dot{x}_{\mathrm{ref}}(t) = A_{\mathrm{ref}}x_{\mathrm{ref}}(t) + B_{\mathrm{ref}}r(t),$$

Solution: Set $u(t) = \hat{K}^{\mathrm{T}}(t)\pi(t, x(t), r(t))$, where

$$\dot{\hat{K}}(t) = -\Gamma\pi(t, x(t), r(t))e^{\mathrm{T}}(t)PB$$

$$\pi(t, x, r) \triangleq \left[x^{\mathrm{T}}, r^{\mathrm{T}}, -\Phi^{\mathrm{T}}(t, x)\right]^{\mathrm{T}},$$

$$e(t) \triangleq x(t) - x_{\mathrm{ref}}(t)$$

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MRAC: Transient Challenges

- "Large" Γ needed for fast adaptation to rapidly varying $r(t)$
- Tracking error dynamics is as fast as reference model
    - Reference models' transient dynamics is missed

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Two-Layer MRAC

Theorem

Introduce reference model for the transient

$$\dot{e}_{\mathrm{ref,transient}}(t) = A_{\mathrm{transient}}\, e_{\mathrm{ref,transient}}(t)$$

s.t. $\mathrm{Re}(\lambda_{\max}(A_{\mathrm{transient}})) < \mathrm{Re}(\lambda_{\min}(A_{\mathrm{ref}}))$. Set
$u(t) = \hat{K}^{\mathrm{T}}(t)\pi(t, x(t), r(t)) + \hat{K}_g^{\mathrm{T}}(t)e(t)$,

$$\dot{\hat{K}}(t) = -\Gamma\pi(t, x(t), r(t))\varepsilon^{\mathrm{T}}(t)P_{\mathrm{transient}}B,$$
$$\dot{\hat{K}}_g(t) = -\Gamma_g e(t)\varepsilon^{\mathrm{T}}(t)P_{\mathrm{transient}}B,$$
$$\varepsilon(t) \triangleq e(t) - e_{\mathrm{ref,transient}}(t)$$

Then, $\lim_{t\to\infty} e(t) = 0$ & $\alpha_{\max}(e(\cdot)) \geq -\mathrm{Re}(\lambda_{\max}(A_{\mathrm{ref}}))$

Control of Quad-Biplanes

Design controller for VTOL UAV

- Uncertainties in aerodynamic models
- Lack of control surfaces
- Nonlinear dynamics

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Tailsitter 6-DOF unified control system

- Design unified control system for tailsitters;
  - Does not distinguish among flight modes
  - Does not distinguish between lateral and longitudinal dynamics
- Two-layer MRAC framework;
- Avoids unwinding by leveraging barrier functions;
- Novel method for deducing reference angular velocities.

Problem Setup

- Four orthonormal reference frames;
  - Inertial: $\mathbb{I} = \{O; X, Y, Z\}$,
  - Body: $\mathbb{J} = \{C(\cdot); x_{\mathbb{J}}(\cdot), y_{\mathbb{J}}(\cdot), z_{\mathbb{J}}(\cdot)\}$,
  - Wind: $\mathbb{W} = \{C(\cdot); x_{\mathbb{W}}(\cdot), y_{\mathbb{W}}(\cdot), z_{\mathbb{W}}(\cdot)\}$.
  - Desired: $\mathbb{K} = \{r_{\mathrm{ref}}(\cdot); x_{\mathbb{K}}(\cdot), y_{\mathbb{K}}(\cdot), z_{\mathbb{K}}(\cdot)\}$.
- Euler parameters used to capture orientation: $q(t) \triangleq [q_1(t), q_{\mathrm{v}}^{\mathrm{T}}(t)]^{\mathrm{T}}$.
- Tracking error quaternion: $q_e(t) \triangleq q^{-1}(t) * q_{\mathrm{ref}}(t)$.
- We want $\mathbb{J}$ to track $\mathbb{K}$ & avoid the unwinding phenomenon.

Dynamic Equations

- Translational equations of motion

$$\ddot{r}_C^{\mathbb{I}}(t) = \frac{1}{m}\left[F_{\mathrm{T}}^{\mathbb{I}}(t) + mg\mathbf{e}_{3,3} + F_{\mathrm{A}}^{\mathbb{I}}(t, p(t), p_{\mathrm{dot}}(t))\right],$$

- Mass $m > 0$ is unknown, $F_{\mathrm{T}}(t) \triangleq u_1(t)\mathbf{e}_{1,3}$ & $F_{\mathrm{A}}^{\mathbb{I}}(\cdot)$ is aerodynamic force.
- Rotational dynamic equations

$$\dot{\omega}(t) = I^{-1}\left[M_{\mathrm{T}}(t) + M_{\mathrm{A}}(t, p(t), p_{\mathrm{dot}}(t)) - \omega^{\times}(t)I\omega(t)\right],$$

- I is matrix of inertia, $M_{\mathrm{T}} \triangleq [u_2(t), u_3(t), u_4(t)]^{\mathrm{T}}$ is moment of the thrust force & $M_{\mathrm{A}}(\cdot)$ is aerodynamic moment

Control Design

- Quadbiplanes are underactuated, cannot control x, y-position directly.
- The control system is separated into an outer and inner loop.
  - Outer loop computes:
    - Ideal thrust;
    - Reference attitude;
    - Reference angular rates to track user-defined trajectory.
  - Inner loop computes:
    - moment of the thrust force;
    - thrust allocation.

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Outer Loop: Thrust Force Determination

- If both the direction and magnitude of the thrust force could be set arbitrarily

$$F_{\mathrm{T,ideal}} = u_{\mathrm{outer}} - R_{\mathbb{W}}^{\mathbb{J}}(\alpha, \beta)[\mathbf{e}_{1,3}, \mathbf{e}_{2,3}, 0_3]\tilde{F}_{\mathrm{A}}^{\mathbb{W}}(t, p, p_{\mathrm{dot}}),$$

where $\tilde{F}_{\mathrm{A}}^{\mathbb{W}}$ is estimate of the aerodynamic force & u_{outer} is the virtual control input so that $\lim_{t\to\infty} \left\| \begin{bmatrix} r_C(t) \\ \dot{r}_C(t) \end{bmatrix} - \begin{bmatrix} r_{\mathrm{user}}(t) \\ \dot{r}_{\mathrm{user}}(t) \end{bmatrix} \right\| = 0$

- $u_1(t)$ must be positive. Thurst direction set by steering the vehicle's attitude.

$$F_{\mathrm{T,proj}}(t) \triangleq \left[\begin{bmatrix} H(F_{\mathrm{T,ideal}}^{\mathrm{T}}(t)\mathbf{e}_{1,3}) \\ 0_2 \end{bmatrix}, \mathbf{e}_{2,3}, \mathbf{e}_{3,3} \right] F_{\mathrm{T,ideal}}(t)$$

Outer Loop: Reference attitude

- Let

$$R_{\mathbb{J}}^{\mathbb{K}}(\tilde{q}_{\mathrm{ref}}(t)) \triangleq u_1(t)\mathbf{e}_{1,3}F_{\mathrm{T,proj}}^{\mathrm{T}}(t) = U(t)\tilde{\Sigma}(t)V^{\mathrm{T}}(t)$$

- $\tilde{\Sigma}(t) \triangleq \mathrm{diag}\{1, 1, \det(U(t)V^{\mathrm{T}}(t))\}$,
- $U, V : [t_0, \infty) \to \mathbb{R}^{3\times 3}$ contain left & right singular vectors,

$$u_1(t) = H(\|F_{\mathrm{T,proj}}(t)\| - T_{\min})\|F_{\mathrm{T,proj}}(t)\|$$

with $T_{\min} > 0$ minimum thrust force

- We guarantee that

$$R_{\mathbb{J}}^{\mathbb{K}}(\tilde{q}_{\mathrm{ref}}(t)) = \mathrm{argmin}_{R\in SO(3)}\|u_1(t)R\mathbf{e}_{1,3} - F_{\mathrm{T,proj}}(t)\|$$

- Orthogonal Procustes problem

Outer Loop: Reference angular rates

- $\tilde{q}_{\mathrm{ref}}(\cdot)$ underlying $R_{\mathbb{K}}^{\mathbb{I}}(\tilde{q}_{\mathrm{ref}})$ is not continuously differentiable. Hence,

$$\omega_{\mathrm{ref}}(t) \triangleq 2J^{\mathrm{T}}(\tilde{q}_{\mathrm{ref}}(t))\dot{\tilde{q}}_{\mathrm{ref}}(t)$$

  cannot be computed directly.

- Given geodesic curve

$$\mathrm{slerp}(p,q,h) \triangleq p * \left(p^{-1} * q\right)^{h}, \qquad (p,q,h) \in \mathbb{H} \times \mathbb{H} \times [0,1].$$

- It holds that

$$\frac{\mathrm{d}}{\mathrm{d}h}\mathrm{slerp}(p,q,h) = \mathrm{slerp}(p,q,h) * \log(p^{-1} * q), \qquad h \in [0,1],$$
$$\frac{\mathrm{d}^2}{\mathrm{d}h^2}\mathrm{slerp}(p,q,h) = -\theta_{\mathrm{slerp}}^2\mathrm{slerp}(p,q,h),$$

Outer Loop: Reference angular rates

- Find $h : [0, t] \to [0, 1]$ s.t.

$$\dot{q}_{\text{ref}}(t) = J(q_{\text{ref}}(t)) \left[\log(q^{-1}(t) * q_{\text{ref}}(t))\right]_{\text{v}} \left.\frac{\mathrm{d}h(\tau)}{\mathrm{d}\tau}\right|_{\tau=t},$$

$$\ddot{q}_{\text{ref}}(t) = J(q_{\text{ref}}(t)) \left[\log(q^{-1}(t) * q_{\text{ref}}(t))\right]_{\text{v}} \left.\frac{\mathrm{d}^2 h(\tau)}{\mathrm{d}\tau^2}\right|_{\tau=t} - \theta^2_{\text{slerp}}(t) q_{\text{ref}}(t) \left(\left.\frac{\mathrm{d}h(\tau)}{\mathrm{d}\tau}\right|_{\tau=t} \right)^2,$$

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Outer Loop: Deducing $\frac{dh(\tau)}{d\tau}$ and $\frac{d^2h(\tau)}{d\tau^2}$

- MPC Problem: Minimize

$$\sum_{i=1}^{n_t} \begin{bmatrix} \Omega_{\mathrm{ref},i} \\ \dot{\Omega}_{\mathrm{ref},i} \end{bmatrix}^{\mathrm{T}} \Theta \begin{bmatrix} \Omega_{\mathrm{ref},i} \\ \dot{\Omega}_{\mathrm{ref},i} \end{bmatrix}$$

s.t.

$$\begin{aligned}
H_{i+1} &= H_i + H_{i,\mathrm{dot}}\delta t + \frac{1}{2}H_{i,\mathrm{ddot}}\delta t^2, \quad i \in \{1,\ldots,n_t-1\},\\
H_{i+1,\mathrm{dot}} &= H_{i,\mathrm{dot}} + H_{i,\mathrm{ddot}}\delta t, \quad i \in \{1,\ldots,n_t-1\},\\
\Omega_{\mathrm{ref},i} &= 2[\log(q^{-1}(t) * q_{\mathrm{ref}}(t))]_{\mathrm{v}} H_{i,\mathrm{dot}}, \quad i \in \{1,\ldots,n_t\},\\
\dot{\Omega}_{\mathrm{ref},i} &= 2[\log(q^{-1}(t) * q_{\mathrm{ref}}(t))]_{\mathrm{v}} H_{i,\mathrm{ddot}}, \quad i \in \{1,\ldots,n_t\},\\
H_1 &= 0,\\
H_{n_t} &= 1,\\
H_i &\in [0,1], \quad i \in \{2,\ldots,n_t-1\},
\end{aligned}$$

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MRAC System Design

- Let $\hat{I} \in \mathbb{R}^{3\times3}$ be an estimate of the matrix of inertia I
- $\tilde{M}_{\rm A}(t, p, p_{\rm dot})$ be an estimate of the moment of the aerodynamic force

$$M_{\rm T}(t) = u_{\rm inner}(t) + \omega^{\times}(t)\hat{I}\omega(t) - \tilde{M}_{\rm A}(t, p(t), p_{\rm dot}(t))$$

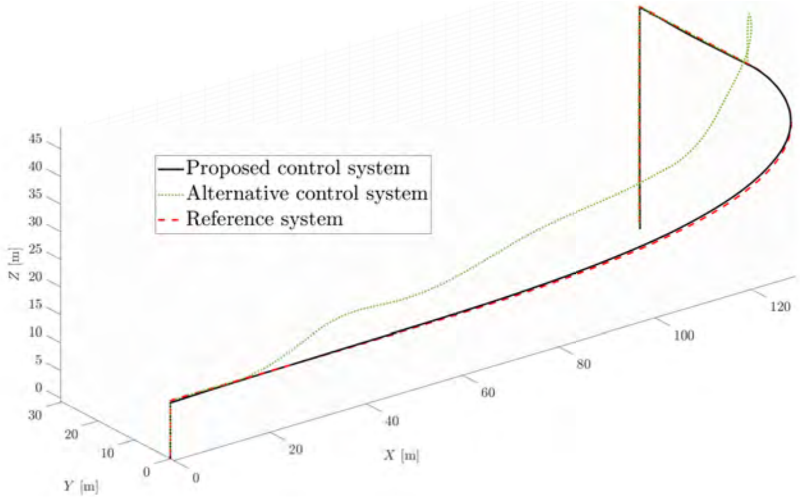
- The equations of motion reduce to

$$\dot{x}_C(t) = A_C x_C(t) + B_C \Lambda_C \left[u^{\mathbb{I}}_{\rm outer}(t) + \Theta^{\rm T}_{\rm outer}\Phi_{\rm outer}(t, p(t), p_{\rm dot}(t))\right]$$
$$\dot{\omega}(t) = I^{-1}\left[u_{\rm inner}(t) + \Theta^{\rm T}_{\rm inner}\Phi_{\rm inner}(t, p(t), p_{\rm dot}(t))\right]$$

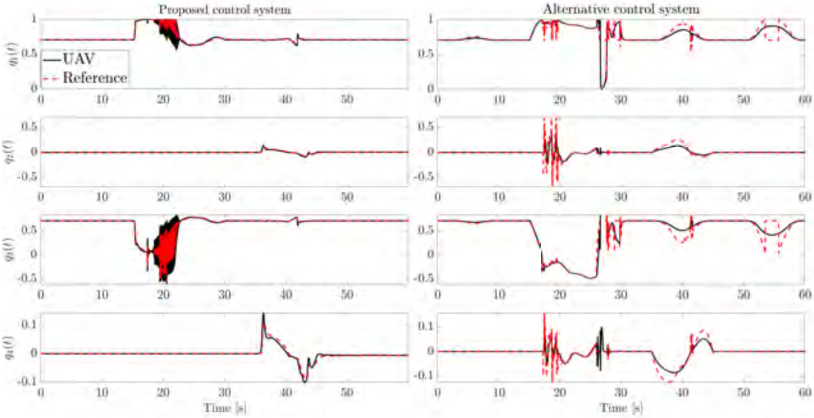
where $x_C(t) \triangleq \left[\left(r^{\mathbb{I}}_C(t)\right)^{\rm T}, \left(\dot{r}^{\mathbb{I}}_C(t)\right)^{\rm T}\right]^{\rm T}$.

- MRAC form!

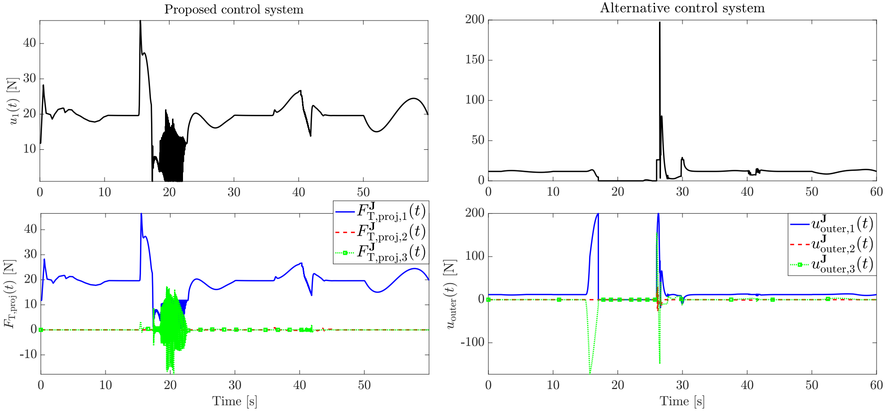
Numerical simulations: Path



Numerical simulations: Attitude



Numerical simulations: Thrust



Preliminary Flight Test: Classical MRAC

Link

Project 3: MRAC for Switched Systems

Find a control law for the plant

$$\dot{x}(t) = A_{\sigma(t)} x(t) + B_{\sigma(t)} \left[u(t) + \Theta^{\mathrm{T}} \Phi_{\sigma(t)}(t, x(t)) \right],$$

where $A_s \in \mathbb{R}^{n \times n}$ & $\Theta \in \mathbb{R}^{N \times m}$ are unknown so that $\lim_{t \to \infty} \|x(t) - x_{\mathrm{ref}}(t)\| = 0$,

$$\dot{x}_{\mathrm{ref}}(t) = A_{\mathrm{ref},\sigma(t)} x_{\mathrm{ref}}(t) + B_{\mathrm{ref},\sigma(t)} r(t),$$

Note

- Same framework can be employed for $\Theta_{\sigma(t)}$
- Switching time $\sigma(\cdot)$ is known w.l.o.g.
  - Uncertainties in $\sigma(\cdot)$ embedded in A & Θ

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Carathéodory & Filippov Frameworks

Consider the switched system

$$\dot{x}(t) = f_{\sigma(t)}(t, x(t)),$$

- Carathéodory solution:

$$x(t) = x_0 + \int_{t_0}^{t} f_{\sigma(\tau)}(\tau, x(\tau))\mathrm{d}\tau, \qquad t \geq t_0 \text{ a.e.}$$

- Filippov solution: If $\exists : x : \mathcal{I} \to \mathcal{D}$ absolutely continuous and s.t.

$$\dot{x}(t) \in K[f_{\sigma(t)}](t, x(t)), \qquad t \in \mathcal{I} \text{ a.e.},$$

  where $t_0 \in \mathcal{I}$

$$K[f_s](t,x) \triangleq \bigcap_{\delta>0} \bigcap_{\mu(\mathcal{N})=0} \overline{\text{co}}\left(f_s\left(t, \mathcal{B}_\delta(x) \setminus \mathcal{N}\right)\right), \quad (s,t,x) \in \Sigma \times [t_0, \infty) \times \mathcal{D},$$

  then, $x(\cdot)$ is a *Filippov solution*

Switched MRAC in Carathéodory Framework

Theorem

Consider $u(t) = \phi(\hat{\Theta}(t), \tilde{\Phi}_{\sigma(t)}(t, x(t)))$ with

$$\phi(\hat{\Theta}, \tilde{\Phi}_s) = \hat{\Theta}^{\mathrm{T}} \tilde{\Phi}_s,$$

$$\dot{\hat{\Theta}}(t) = -\Gamma \tilde{\Phi}_{\sigma(t)}(t, x(t)) e^{\mathrm{T}}(t) P B_{\sigma(t)},$$

$$\tilde{\Phi}_s(t, x) \triangleq \begin{bmatrix} \mathcal{I}(s) \otimes x \\ \mathcal{I}(s) \otimes r(t) \\ -\Phi_s(t, x) \end{bmatrix},$$

$$\mathcal{I}(s) \triangleq \left[\mathbf{1}_{\{s \in \Sigma : s - 1 = 0\}}(s), \ldots, \mathbf{1}_{\{s \in \Sigma : s - \overline{\sigma} = 0\}}(s)\right]^{\mathrm{T}}$$

Switched MRAC in Carathéodory Framework (cont'd)

Theorem (cont'd)

If dwell time $t_{\mathrm{d}} > 0$ & $\exists$ symmetric P.D. matrices $P, Q \in \mathbb{R}^{n \times n}$ s.t.

$$A_{\mathrm{ref},s}^{\mathrm{T}} P + P A_{\mathrm{ref},s} < -Q, \qquad s \in \Sigma$$

Then, both $e(\cdot)$ & $\hat{\Theta}(\cdot)$ are bounded uniformly in both $t_0 \in [0, \infty)$ & $\sigma(\cdot)$, & $e(t) \to 0$ as $t \to \infty$ uniformly in both t_0 and $\sigma(\cdot)$

Remark

To prove this result, we extended the LaSalle-Yoshizawa theorem to switched systems in Caratéodory framework

Switched MRAC in Filippov Framework

Theorem

Consider $u(t) = \phi(\hat{\Theta}(t), \tilde{\Phi}_{\sigma(t)}(t, x(t)))$ with $\sigma(\cdot)$ Lebesgue integrable

$$\begin{aligned}\phi(\hat{\Theta}, \tilde{\Phi}_s) &= \hat{\Theta}^{\mathrm{T}} \tilde{\Phi}_s, \\ \dot{\hat{\Theta}}(t) &= -\Gamma \tilde{\Phi}_{\sigma(t)}(t, x(t)) e^{\mathrm{T}}(t) P B_{\sigma(t)}, \\ \tilde{\Phi}_s(t, x) &\triangleq \begin{bmatrix} \mathcal{I}(s) \otimes x \\ \mathcal{I}(s) \otimes r(t) \\ -\Phi_s(t, x) \end{bmatrix},\end{aligned}$$

Both $e(\cdot)$ & $\hat{\Theta}(\cdot)$ are bounded uniformly in $t_0 \in [0, \infty)$ & $\sigma(\cdot)$, & $e(t) \to 0$ as $t \to \infty$ uniformly in t_0 and $\sigma(\cdot)$

Note: $t_{\mathrm{d}} = 0$ is allowed! Uniqueness of solution *not* guaranteed.

UAVs for Sensor Mounting

Design controller for Tiltrotor UAV

- Switched dynamical models
- Parametric Uncertainties
- Nonlinear dynamics

UAVs for Sensor Mounting

Equations of motion

Switching between tilt-rotor mode & cantilever beam modes

$$\mathcal{H}_{\sigma(t)}\mathcal{M}(q(t))\begin{bmatrix}\dot{v}_A^{\mathbb{I}}(t)\\ \dot{\omega}(q(t),\dot{q}(t))\end{bmatrix}=\mathcal{H}_{\sigma(t)}\left(\begin{bmatrix}f_{\rm dyn,tran}(t,q(t),\dot{q}(t))\\ f_{\rm dyn,rot}(t,q(t),\dot{q}(t))\end{bmatrix}+G(q(t))u(t)\right),$$

where

$$\mathcal{H}_s\triangleq\begin{bmatrix}\mathbf{1}_{\{s\in\Sigma:s-1=0\}}(s)I_3 & 0_{3\times3}\\ 0_{3\times3} & \mathcal{I}_{\rm rot}(s)\end{bmatrix},\qquad \mathcal{M}(q)\triangleq\begin{bmatrix}mI_3 & -mR(q)r_C^{\times}\\ mr_C^{\times}R^{\rm T}(q) & \mathfrak{I}\end{bmatrix},$$

$$f_{\rm dyn,tran}(t,q,\dot{q})\triangleq F_g^{\mathbb{I}}-mR(q)\omega^{\times}(q,\dot{q})\omega^{\times}(q,\dot{q})r_C,$$

$$f_{\rm dyn,rot}(t,q,\dot{q})\triangleq-\omega^{\times}(q,\dot{q})\mathfrak{I}\omega(q,\dot{q})-\sum_{i=1}^{4}\left[\mathfrak{I}_{P_i}(t)\dot{\omega}_{P_i}(t)+\omega_{P_i}^{\times}(t)\mathfrak{I}_{P_i}(t)\omega_{P_i}(t)\right]$$

$$-\omega^{\times}(q,\dot{q})\sum_{i=1}^{4}\mathfrak{I}_{P_i}(t)\omega_{P_i}(t)+r_C^{\times}R^{\rm T}(q)F_{\rm g}^{\mathbb{I}},$$

$$G(q)\triangleq\begin{bmatrix}R(q)\begin{bmatrix}\mathbf{e}_{1,3} & \mathbf{e}_{3,3}\end{bmatrix} & 0_{3\times3}\\ 0_{3\times2} & I_3\end{bmatrix}$$

UAVs for Sensor Mounting – Flight Tests

Apply feedback-linearizing control law

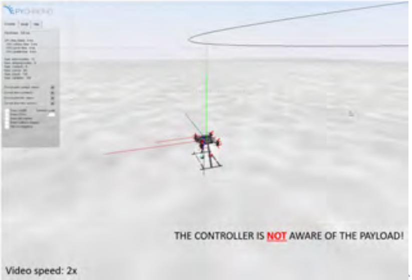
$$\beta_s(t, q, q_{\mathrm{ref}}, w) \triangleq h\Bigg(- \begin{bmatrix} f_{\mathrm{dyn,tran}}(t, q, \dot{q}) \\ f_{\mathrm{dyn,rot}}(t, q, \dot{q}) \end{bmatrix} + \mathcal{M}(q) \begin{bmatrix} \mathbf{1}_3 & 0_{3\times 3} \\ 0_{3\times 3} & \Gamma^{-1}(q) \end{bmatrix} \cdot \left[\ddot{q}_{\mathrm{ref}} - \begin{bmatrix} 0_{3\times 1} \\ \dot{\Gamma}(q)\omega(q, \dot{q}) \end{bmatrix} - [K_{\mathrm{P},s}, K_{\mathrm{D},s}] \begin{bmatrix} q - q_{\mathrm{ref}} \\ \dot{q} - \dot{q}_{\mathrm{ref}} \end{bmatrix} + w \right] \Bigg),$$

Apply switched MRAC law to design w

Link

Ongoing Research

- Hybrid MRAC & UAV Control
  - Unsteady payload delivery
  - Improved tracking performance
- UAVs in caves and mines
  - Very challenging environment (unstructured, windy, dark,...)
  - UAVs to explore mine in visible range & with ground penetrating radar
- Integrated guidance (MPC) & control (MRAC)
- MRAC on infinite-dimensional spaces
  - What if we do not know anything about functional uncertainties?



We Are Hiring!

Looking for Graduate Students (MS/Ph.D.)

- Motivated
- Interested in
  - Control theory/Applied math
  - Robotics
  - Matlab & C++ or Python
- Topics:
  - Hybrid systems
  - Very high-fidelity simulations
  - Guidance & control integration
  - VTOL UAVs for Shipboard Landing
  - UAV & UGV formations in contested environments

Thanks to Students

Former Ph.D. Students

- Dr. Robert B. Anderson
- Dr. Julius A. Marshall

M.S. Students

- Shardul Amrite -- M.S. in ME
- Grant I. Carter -- M.S. in ME
- Rishabh Chaure -- M.S. in ME
- Sean E. Eagen -- M.S. in ME
- Christopher H. Miller -- M.S. in ME
- Japnit Sethi -- M.Eng. in ECE
- Timothy A. Blackford -- M.S. in AE
- John-Paul P. Burke -- M.S. AE
- Coleton Domann -- M.S. AE
- Joshua Karinshak -- M.S. ME
- Soni Ravi -- M.S. ECE

Current Graduate Students

- Mattia Gramuglia – ME, Ph.D.
- Giri Mugundan Kumar – ME, Ph.D.
- Jyotirmoy Mukherjee – ME, Ph.D.
- Haoran Wang – ME, Ph.D.
- Hannah White – ME, Ph.D.
- Paul Binder – AE, MS
- Brian Scurlock – ME, MS
- Sanjana Sanjay Dhulla – CS, MS
- Soha Kshirsagar – CS, MS
- Hudson Smith – MS, ME
- Hyndavi Venkatreddygari – ECE, MEng

Selected B.S. Students

- Siddart Ashok – AE
- Casper Gleich – ISE
- Freddie Mistichelli – AE
- Jorge Flores-Portillo – ECE
- Megan Schneider – ISE
- Ryan Pecoraro – AE
- Adam Tyler – ME
- Ryan Gannon – AE
- Lauren Ingmire – AE
- Haydn Kirkpatrick – ME

Advanced Control Systems Lab

Thanks to Funding Agencies & Collaborators

Questions?

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