



**POLITECNICO
DI TORINO**



Identification of Blade-root Joint Dynamics in Turbine Disks

PhD 33rd cycle (2017-2020)

ZEESHAN SAEED

Supervisors

Prof. Teresa M. Berruti

Prof. Christian M. Firrone

Department of Mechanical and Aerospace Engineering

Politecnico di Torino

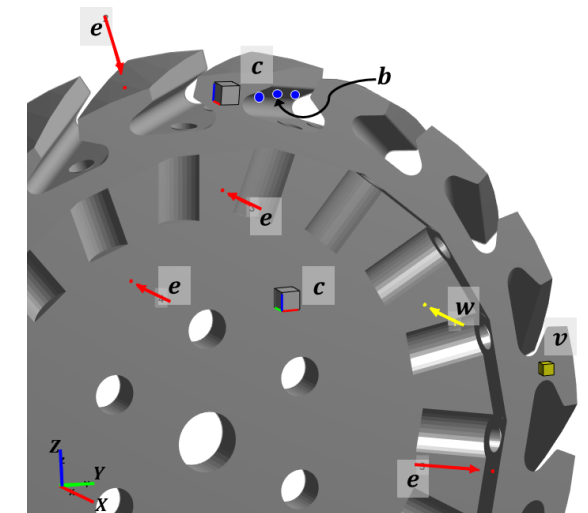
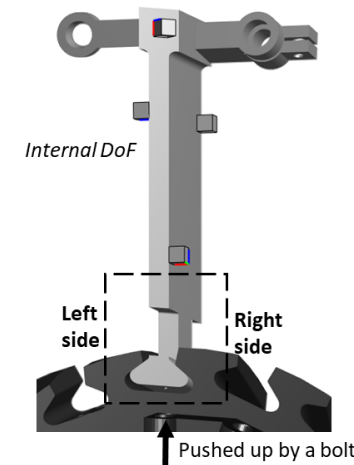
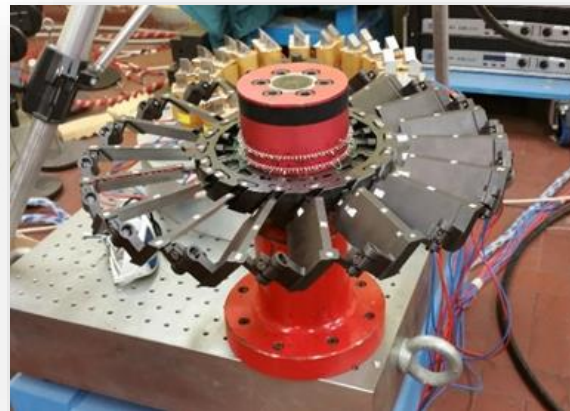
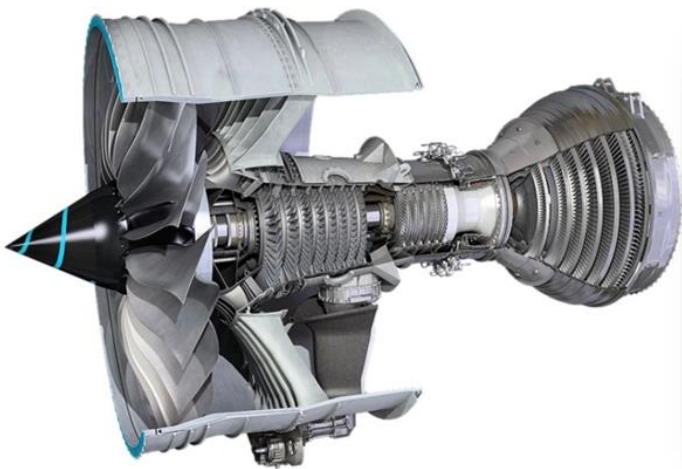
17th Dec. 2020

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- *Introduction*
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- *Motivation*
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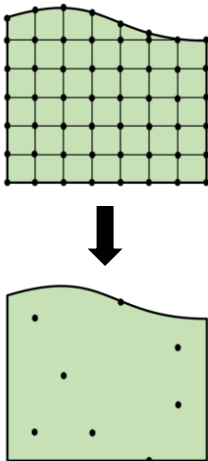
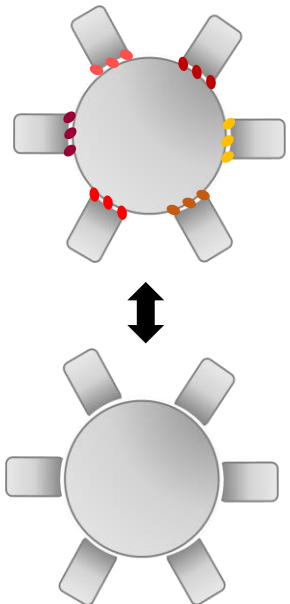
Introduction

- Bladed-disk part of turbomachines
 - Risks: high vibrations, HCF
 - Better response prediction of bladed-disks
 - Identifying the connection dynamics of a blade-root joint
- Given the challenges of such structures
 - inaccessible interface DoF
 - 3-dimensional motion
 - rigid joint
 - VITAL bladed-disk
 - 1 blade assembly
 - Dove-tail type joint



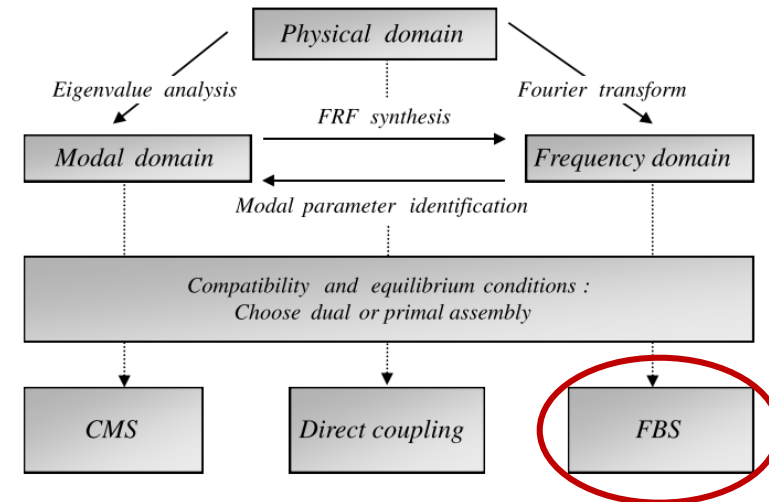
Dynamic Substructuring

- Response prediction of built-up structures
- Dynamic substructuring
 - substructure \rightarrow assembly or vice versa
 - Domain decomposition
 - ROM (needed for analysis, design, optimization)



Rixen 2018

- Substructure representations:
 - Physical or spatial $\mathbf{K}, \mathbf{M}, \mathbf{C}, \mathbf{G}$
 - Modal Φ, Λ
 - Response-based (output/input) $\mathbf{h}(t), \mathbf{H}(\omega)$



De Klerk 2008

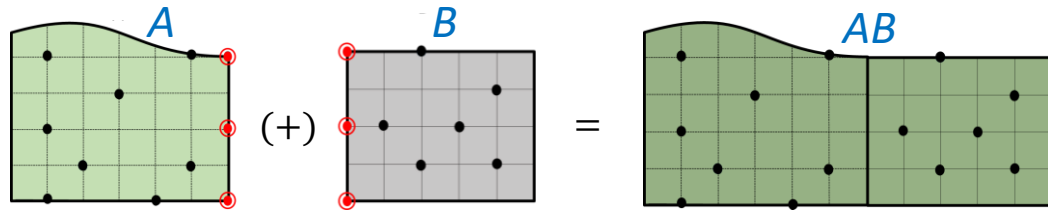
- Experimental FRFs
 - Directly measured
 - Measurement errors
 - Methods sensitive to errors

Frequency based substructuring (FBS)

Coupling (primal or dual)

De Klerk 2006
De Klerk 2008

Rigid connection



$$\mathbf{u} = \begin{Bmatrix} \mathbf{u}^A \\ \mathbf{u}^B \end{Bmatrix} \quad \mathbf{Y} = \begin{bmatrix} \mathbf{Y}^A & \\ & \mathbf{Y}^B \end{bmatrix}$$

$$\mathbf{u} = \mathbf{Y}(\mathbf{f} - \mathbf{B}^T \boldsymbol{\lambda})$$

$$\mathbf{B}\mathbf{u} = \mathbf{0}$$

$$\mathbf{Y}^{AB} = \mathbf{Y} - \mathbf{Y}\mathbf{B}^T(\mathbf{B}\mathbf{Y}\mathbf{B}^T)^{-1}\mathbf{B}\mathbf{Y}$$

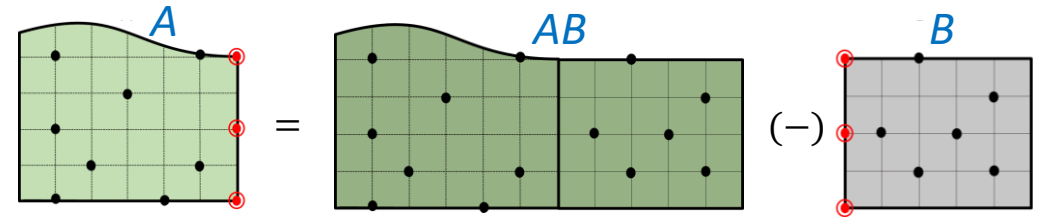
De Klerk 2006

$$\mathbf{Y}^{AB} = fbs(\mathbf{Y}^A, \mathbf{Y}^B)$$

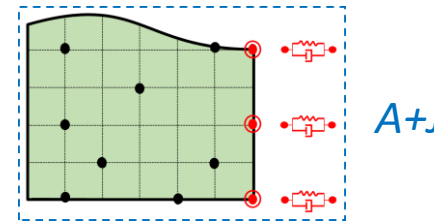
Decoupling

Sjövall 2008
D'Ambrogio 2014

$$\mathbf{Y}^A = fbs(\mathbf{Y}^{AB}, -\mathbf{Y}^B)$$

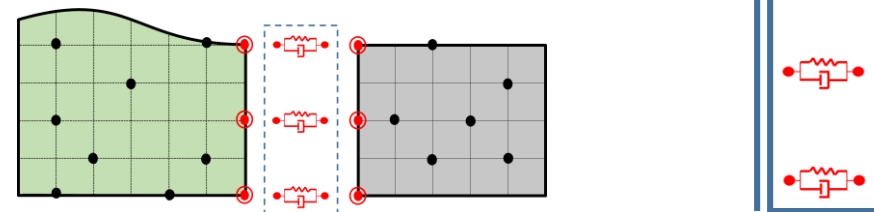


- Flexible connection



- If desired only joint or connection dynamics

$$\mathbf{Y}^J = fbs(\mathbf{Y}^{AJB}, -\mathbf{Y}^B, -\mathbf{Y}^A)$$



Joint Identification by Substructuring

Major Examples in Literature

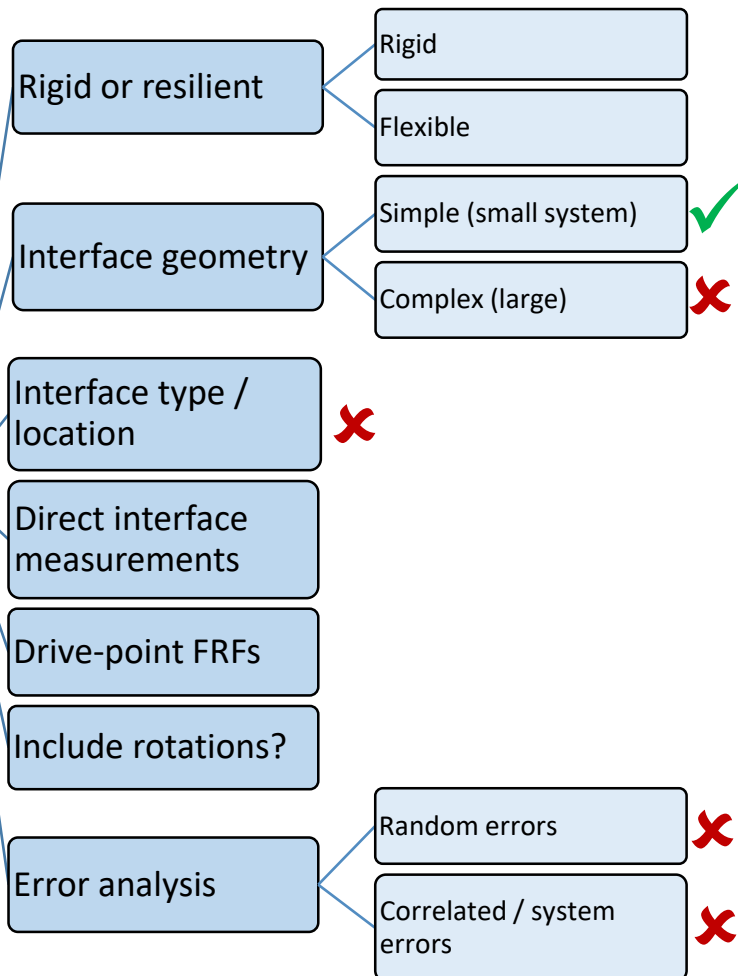
Rigid Joints

- i. A 2×2 spring-damper system on two bolted beams. Substructures by modal filters (*Tsai 1988*)
- ii. A 4×4 spring-damper system on beams (*Ren 1995*)
- iii. A 2×2 rotational DoF joint system with inertia (*Mehrpouya 2013*)
- iv. A 2×2 spring-damper system on two bolted beams (*Tol 2015*)

Resilient Joints

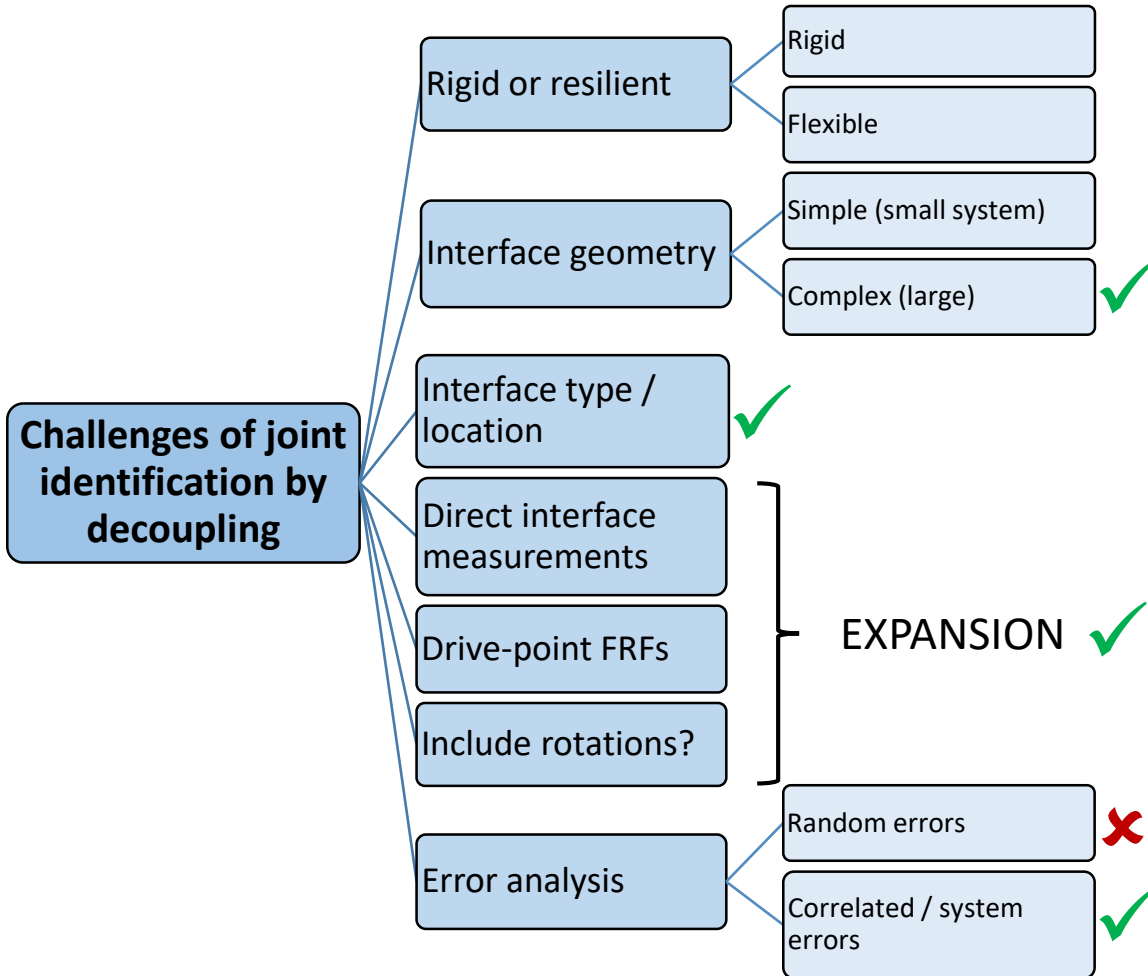
- i. A 6 DoF resilient joint (numerical) and further simplified non-linear (*Keersmaeker 2015*)
- ii. A 6-DoF (12×12 system with mass) resilient element between two rigid crosses (*Haeussler 2020*)

Challenges of joint identification by decoupling



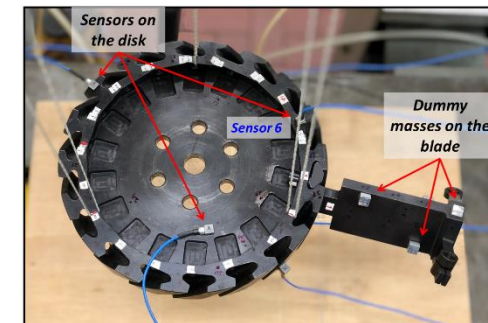
Research objectives and outline

Research problems addressed



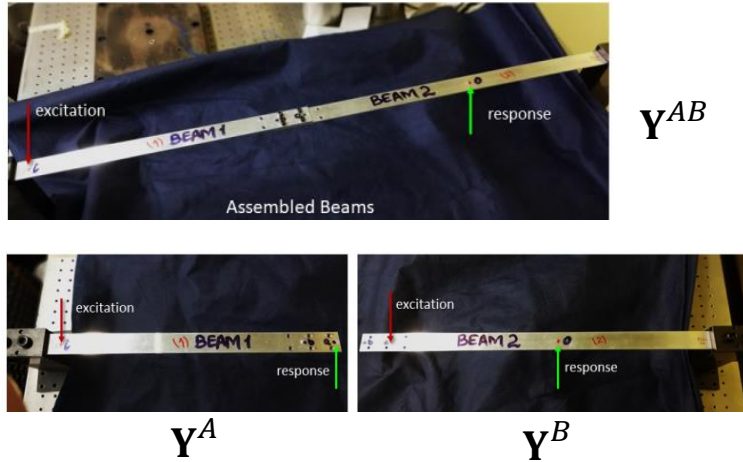
Outline

- Testing a traditional method on a simple assembly of beams [\(Saeed 2019 JPCS\)](#)
- Initial sensitivity studies for blade-root interface system size [\(Saeed 2019 JPCS\)](#)
- Expanded FRFs for blade and disk substructures [\(Saeed 2020 JVA\)](#)
- Apply the expansion based decoupling strategy [\(Saeed 2020 JVA\)](#)
- Proposing a new correlated SEMM approach [\(Saeed 2020 TE, Saeed 2020 JEGTP\)](#)
- Joint identification with correlated SEMM models [\(Saeed 2021 JSV\)](#)
- Parametric improvements in substructures and the identified joint [\(Saeed 2020 ISMA\)](#)



FRF decoupling method (ToI 2015)

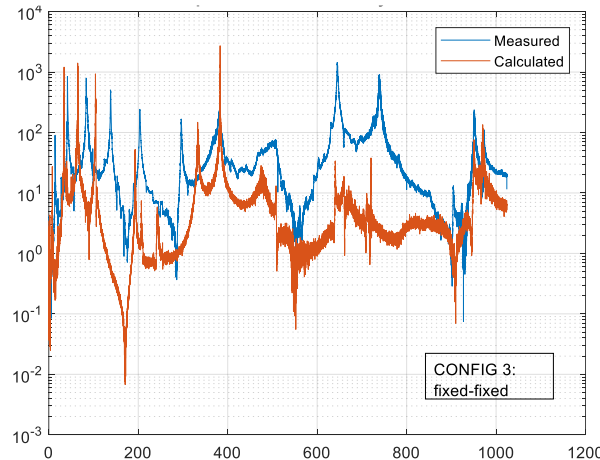
Fully Experimental



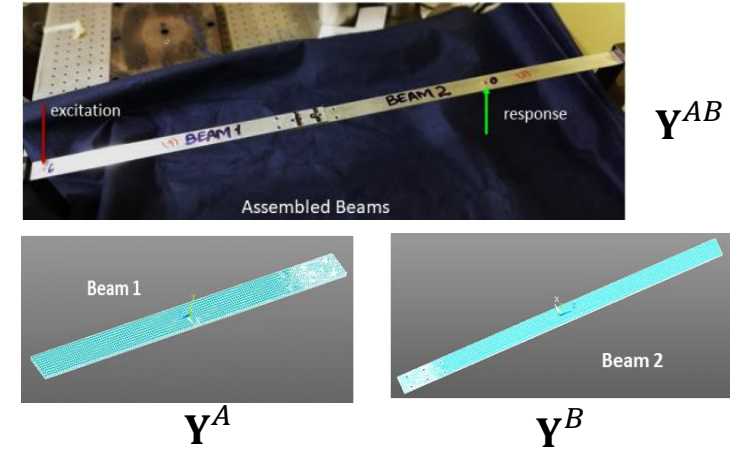
$$\mathbf{Z}_{int}^* = \mathbf{Y}_{bi}^A (\mathbf{Y}_{oi}^{AB})^{-1} \mathbf{Y}_{oc}^B - \mathbf{Y}_{bb}^A - \mathbf{Y}_{bb}^B$$

Simple interface:

- Bolted beams
- Planar motion
- 2 DoF coupling



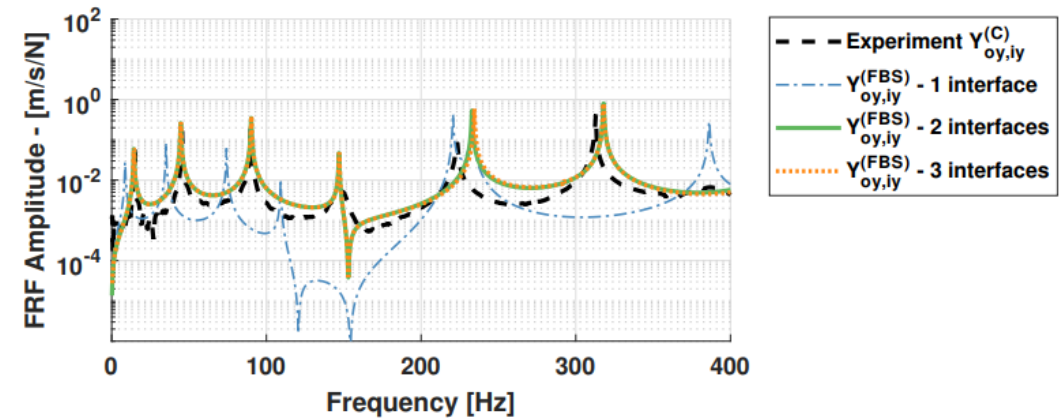
Hybrid



Assembly:
measured FRFs

Substructures:
numerically updated
FRFs

$$\mathbf{Z}_{int}^* = \mathbf{Y}_{bi}^A (\mathbf{Y}_{oi}^{AB})^{-1} \mathbf{Y}_{oc}^B - \mathbf{Y}_{bb}^A - \mathbf{Y}_{bb}^B$$



Substructure models by SEMM

$$\mathbf{u}_c^{\text{ov}} = \mathbf{Y}^{\text{ov}} \mathbf{f}_e^{\text{ov}} \quad \text{where} \quad \mathbf{Y}^{\text{ov}} = \mathbf{Y}_{ce}^{\text{exp}}$$

Overlay model

$$\mathbf{u}_g^{\text{N}} = \begin{Bmatrix} \mathbf{u}_c \\ \mathbf{u}_e \\ \mathbf{u}_o \end{Bmatrix}^{\text{N}} \quad \text{with} \quad \mathbf{u}_o^{\text{N}} = \begin{Bmatrix} \mathbf{u}_v \\ \mathbf{u}_b \end{Bmatrix}^{\text{N}}$$

Numerical model

Removed model

$$\mathbf{Y}^{\text{N}} = \mathbf{Y}_{gg}^{\text{N}} = \begin{bmatrix} \mathbf{Y}_{cc} & \mathbf{Y}_{ce} & \mathbf{Y}_{co} \\ \mathbf{Y}_{ec} & \mathbf{Y}_{ee} & \mathbf{Y}_{eo} \\ \mathbf{Y}_{oc} & \mathbf{Y}_{oe} & \mathbf{Y}_{oo} \end{bmatrix}^{\text{N}}$$

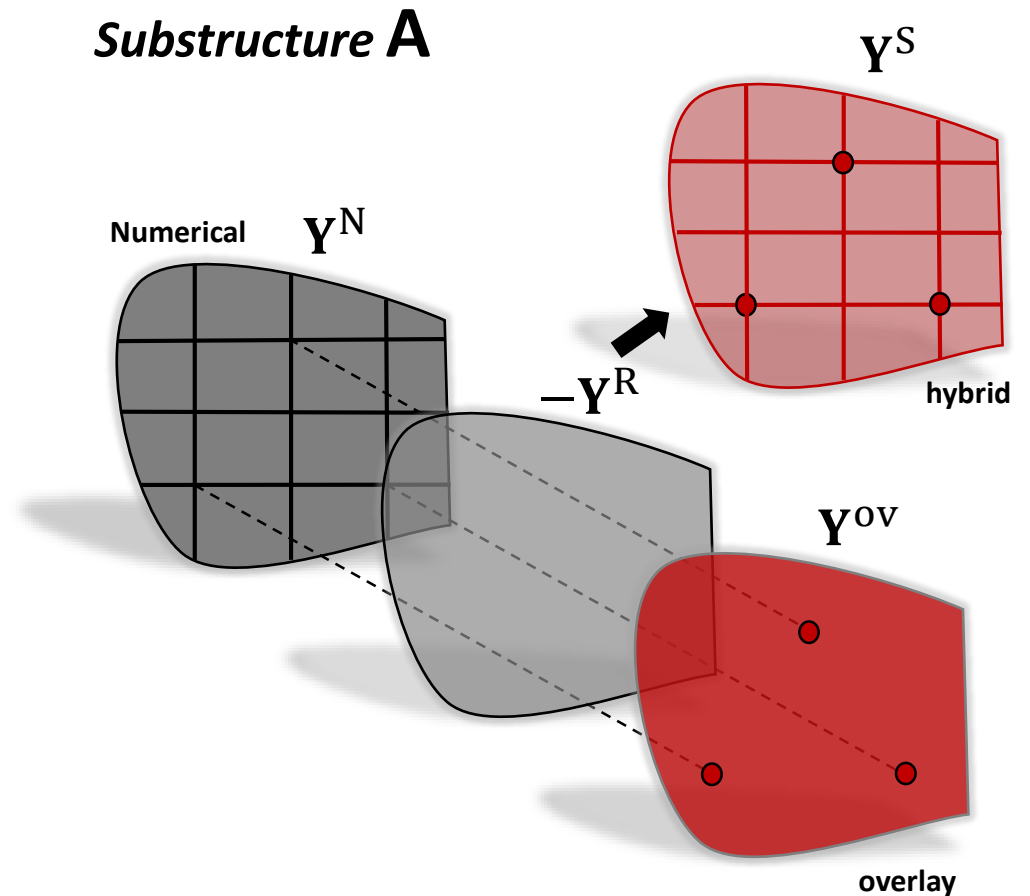
$$\mathbf{Y}^{\text{R}} = \mathbf{Y}^{\text{N}}$$

$$\mathbf{Y}^{\text{S}} = \mathbf{Y}_{gg}^{\text{N}} - \mathbf{Y}_{gg}^{\text{N}} (\mathbf{Y}_{cg}^{\text{N}})^+ (\mathbf{Y}_{ce}^{\text{N}} - \mathbf{Y}_{ce}^{\text{ov}}) (\mathbf{Y}_{ge}^{\text{N}})^+ \mathbf{Y}_{gg}^{\text{N}}$$

$$\mathbf{Y}^{\text{S}} = \text{semm}(\mathbf{Y}^{\text{N}}, \mathbf{Y}^{\text{ov}})$$

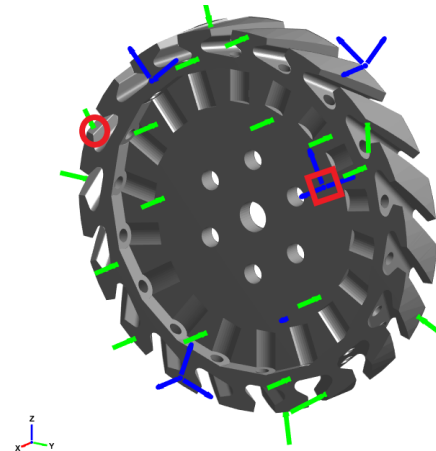
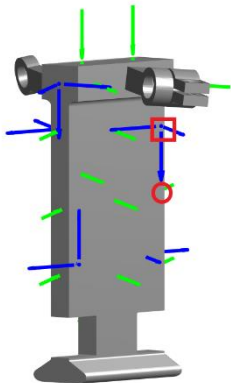
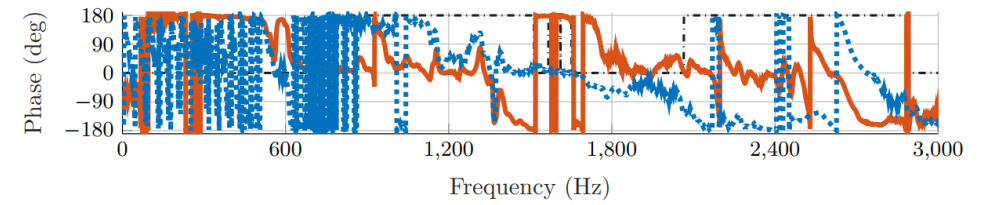
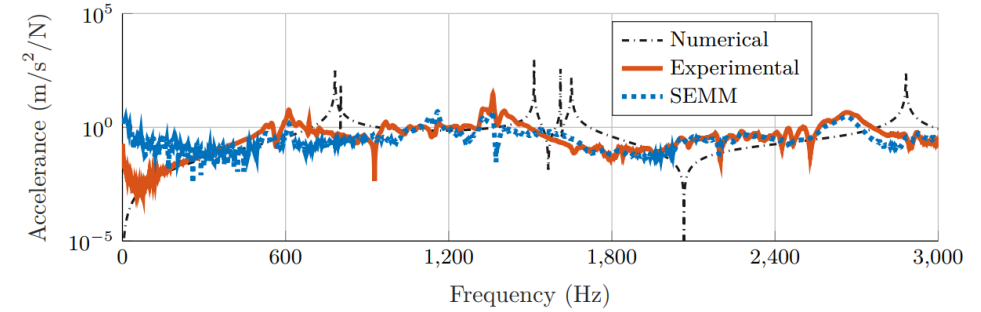
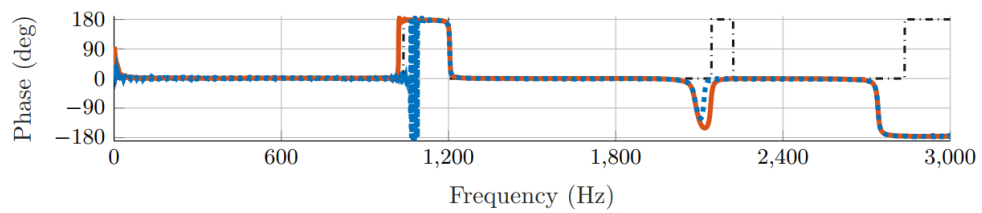
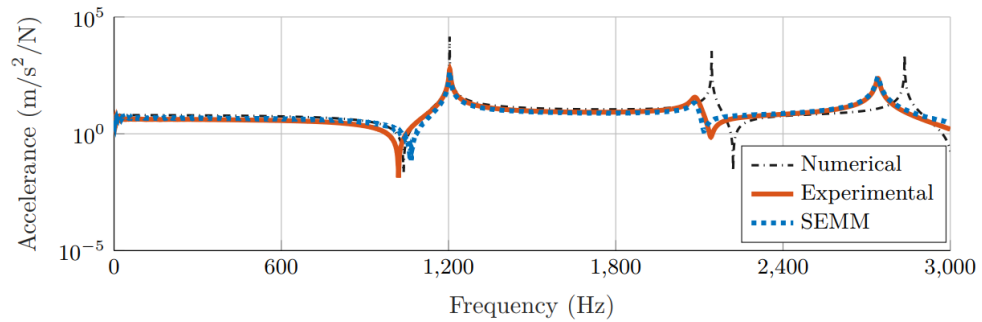
Hybrid or expanded model

Substructure A



SEMM on Substructures

$$\mathbf{Y}^S = \mathbf{Y}_{gg}^N - \mathbf{Y}_{gg}^N (\mathbf{Y}_{cg}^N)^+ (\mathbf{Y}_{ce}^N - \mathbf{Y}_{ce}^{ov}) (\mathbf{Y}_{ge}^N)^+ \mathbf{Y}_{gg}^N$$



Joint identification strategy

$$1 \quad \begin{cases} \mathbf{Y}^{S,A} = \text{semm}(\mathbf{Y}^{N,A}, \mathbf{Y}^{\text{ov},A}) \\ \mathbf{Y}^{S,B} = \text{semm}(\mathbf{Y}^{N,B}, \mathbf{Y}^{\text{ov},B}) \end{cases}$$

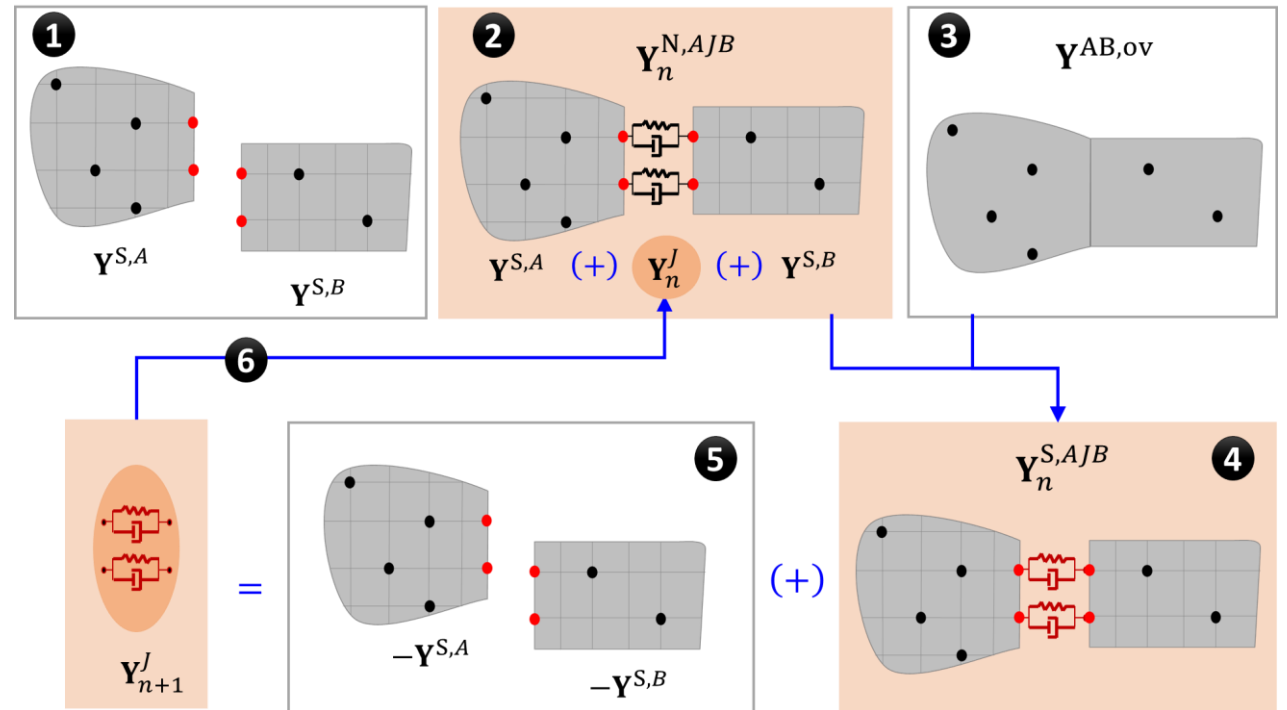
$$2 \quad \mathbf{Y}_n^{N,AJB} = \text{fbs}(\mathbf{Y}^{S,A}, \mathbf{Y}_n^J, \mathbf{Y}^{S,B})$$

$$4 \quad \mathbf{Y}_n^{S,AJB} = \text{semm}(\mathbf{Y}_n^{N,AJB}, \mathbf{Y}^{\text{ov},AB})$$

$$5 \ \& \ 6 \quad \mathbf{Y}_{n+1}^J = \text{fbs}(\mathbf{Y}_n^{S,AJB}, -\mathbf{Y}^{S,A}, -\mathbf{Y}^{S,B})$$

$$\mathbf{Y}^S = \mathbf{Y}_{gg}^N - \mathbf{Y}_{gg}^N (\mathbf{Y}_{cg}^N)^+ (\mathbf{Y}_{ce}^N - \mathbf{Y}^{\text{ov}}) (\mathbf{Y}_{ge}^N)^+ \mathbf{Y}_{gg}^N$$

$$(\mathbf{Y}_{cg}^N)^+ = \mathbf{W} \mathbf{Y}_{cg}^N T \left(\mathbf{Y}_{cg}^N \mathbf{W} \mathbf{Y}_{cg}^N T \right)^{-1}$$



Joint identification of the blade-root

Dummy joint identification

Virtual point interface

$$\begin{Bmatrix} \mathbf{u}_i^A \\ \mathbf{q}^A \end{Bmatrix} = \underbrace{((\mathbf{R}^A)^T \mathbf{R}^A)^{-1} (\mathbf{R}^A)^T}_{\mathbf{T}_u^A} \begin{Bmatrix} \mathbf{u}_i^A \\ \mathbf{u}_b^A \end{Bmatrix}$$

$$\bar{\mathbf{Y}}^{S,A} = \mathbf{T}^A \mathbf{Y}^{S,A} (\mathbf{T}^A)^T$$

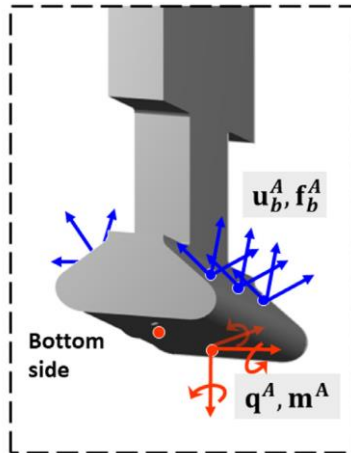
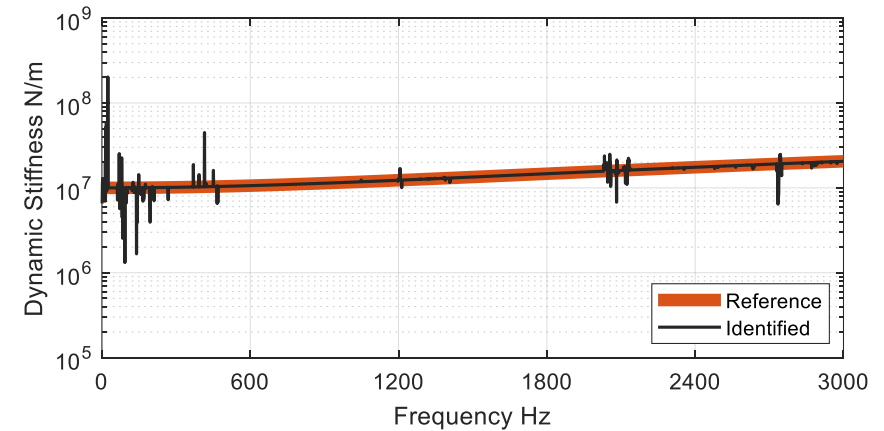


Table 1 Parameters of the dummy joint

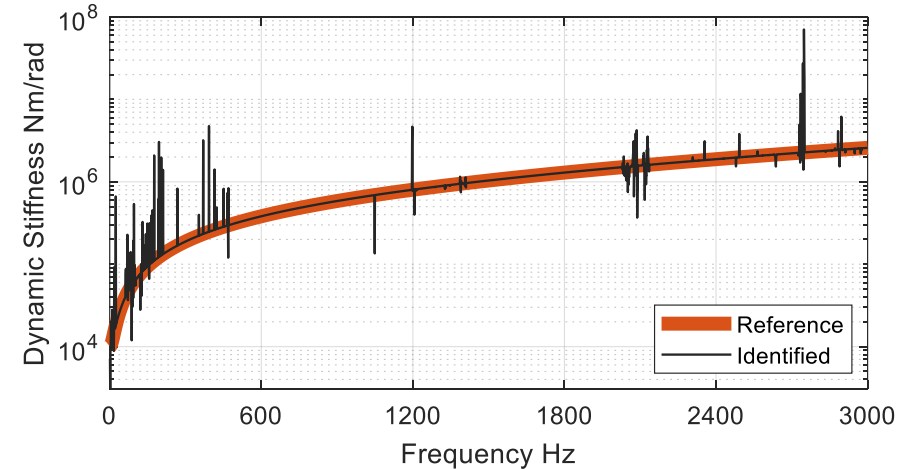
	Translational	Rotational
Stiffness	1×10^7 N/m	1×10^4 Nm/rad
Damping	1×10^3 Ns/m	1×10^2 Nms/rad
Mass	5 g	$5 \text{ gm}^2/\text{rad}$

24 × 24
Joint system

Translational DoF

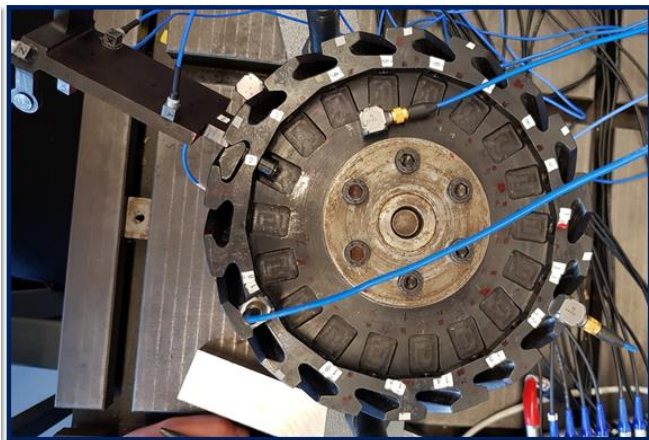
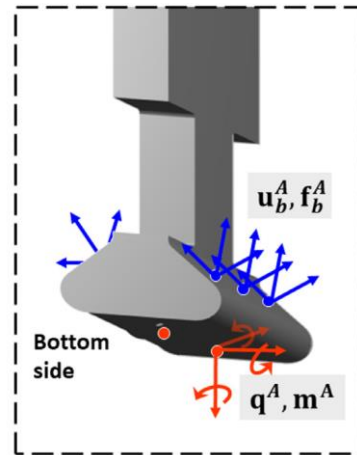
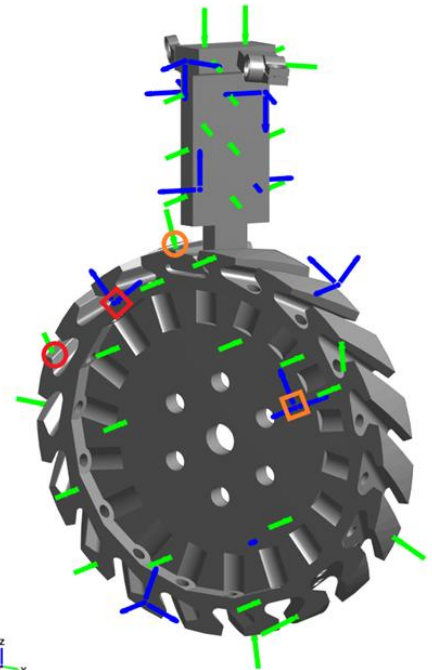


Rotational DoF

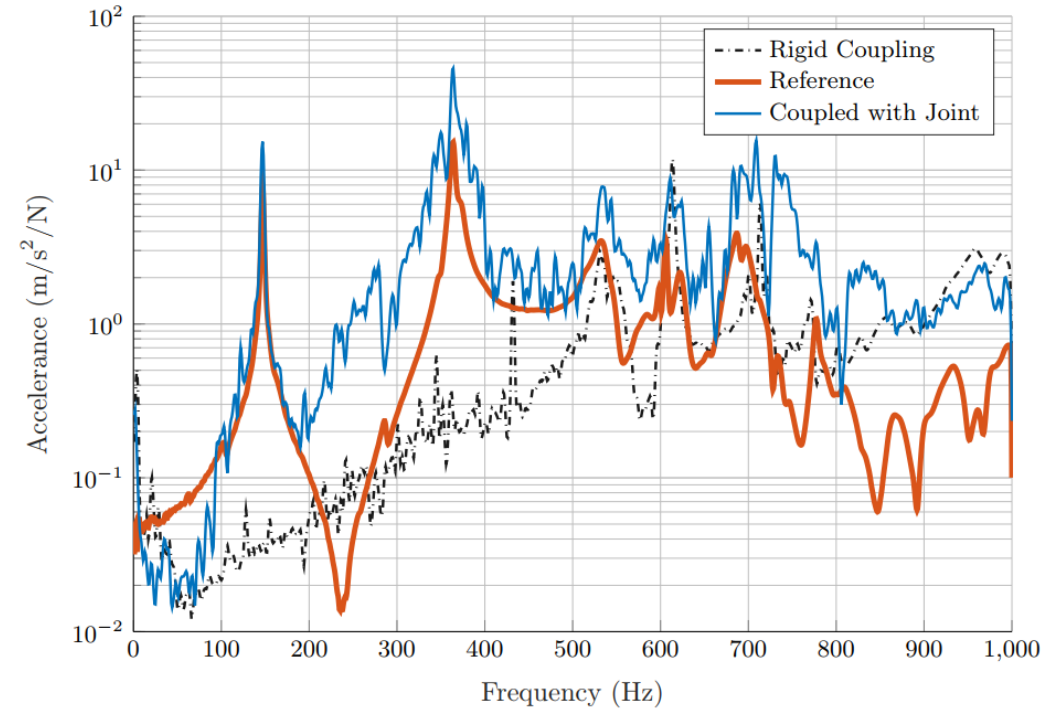


Joint identification of the blade-root

Actual joint identification & Validation



Validation FRF



- *First time possible to validate for, at least the first mode of the assembly to be predicted for such an interface.*
- *Major factor: constraint modelling of the disk*
- *Improvement is required!*

Correlation Strategy in SEMM (Correlated SEMM)

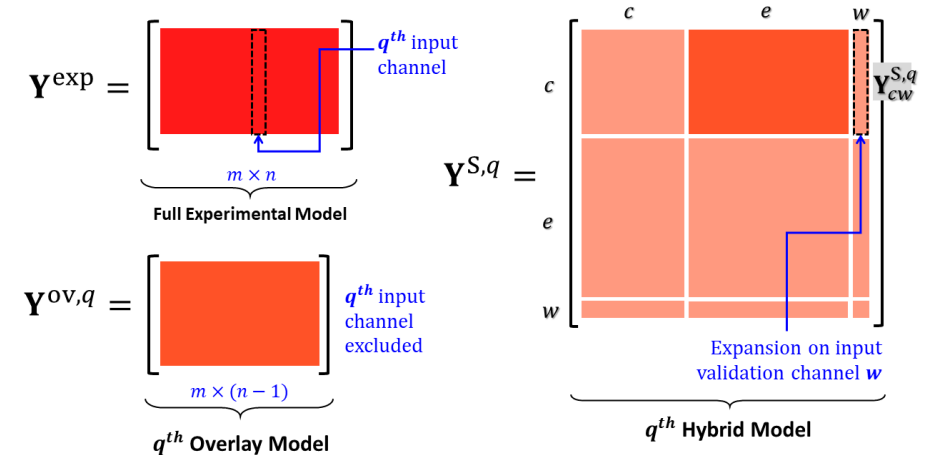
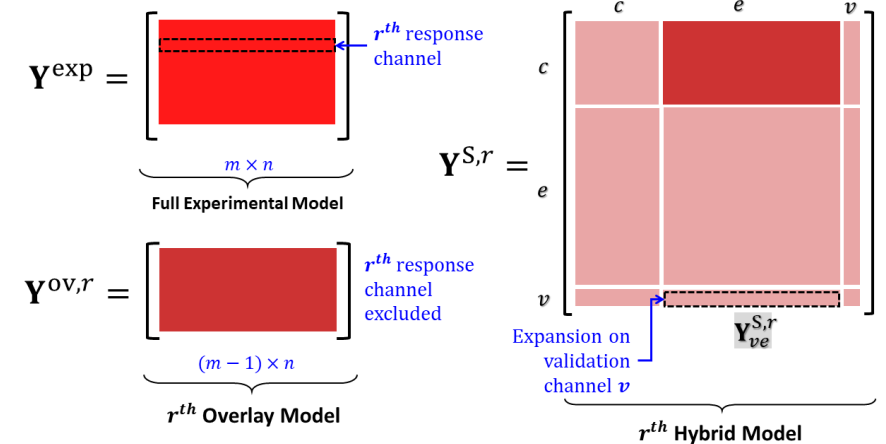
- Propagation of errors in \mathbf{Y}^S
 - Measurement noise
 - Discrepancies between numerical and experimental FRFs
 - Only visual / coherence checks – Online
1. Strictly using $\mathbf{Y}^{ov} \subset \mathbf{Y}_{ce}^{exp}$, i.e. different versions of the overlay model from the same experiments

$$\mathbf{Y}^{ov,r} \subset \mathbf{Y}^{exp} : \mathbf{Y}_{re}^{exp} \notin \mathbf{Y}^{ov,r} \quad r = 1, \dots, m$$

2. Generate hybrid model $\mathbf{Y}^{S,r}$

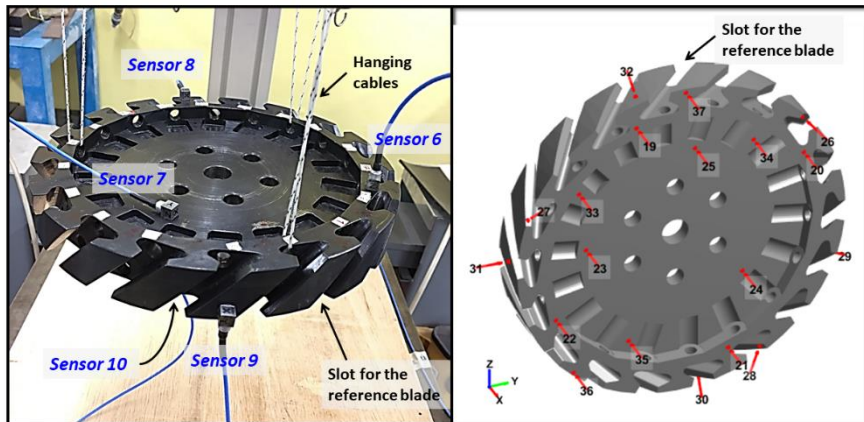
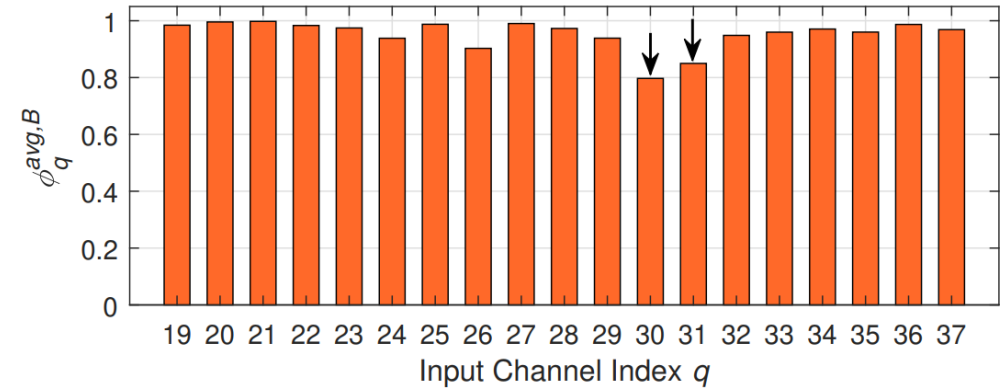
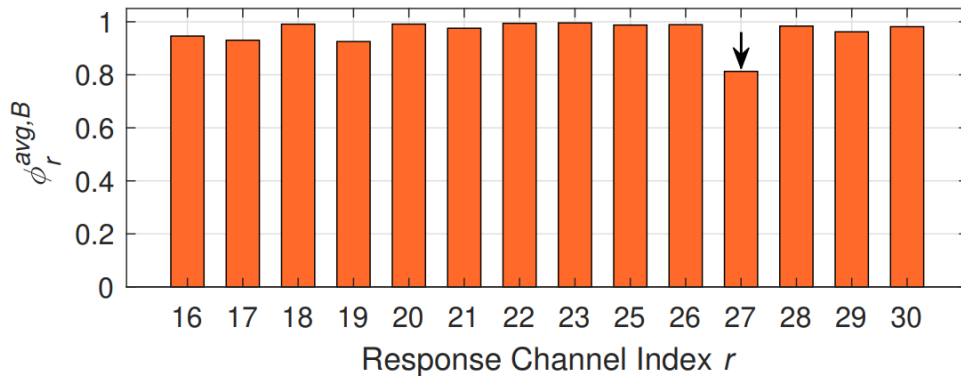
3. Compute FRAC

$$\phi_{rj} = FRAC \left(\mathbf{Y}_{oj}^{S,r}(\omega), \mathbf{Y}_{rj}^{exp}(\omega) \right) = \frac{\left| \mathbf{Y}_{vj}^{S,r}(\omega) \mathbf{Y}_{rj}^{exp*}(\omega) \right|^2}{\mathbf{Y}_{vj}^{S,r}(\omega) \mathbf{Y}_{vj}^{S,r*}(\omega) \cdot \mathbf{Y}_{rj}^{exp}(\omega) \mathbf{Y}_{rj}^{exp*}(\omega)}$$

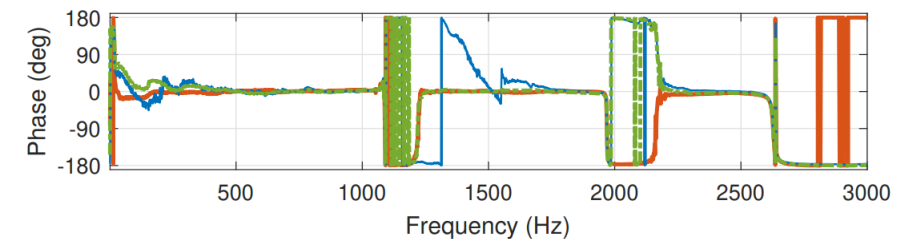
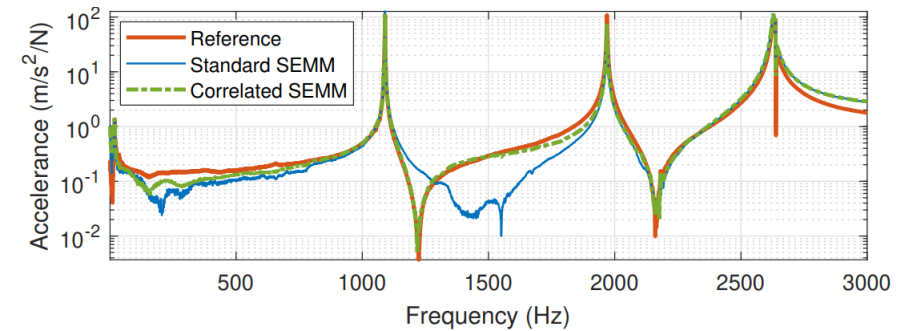


Correlated SEMM on the disk

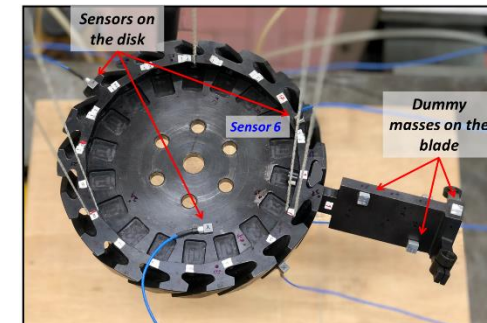
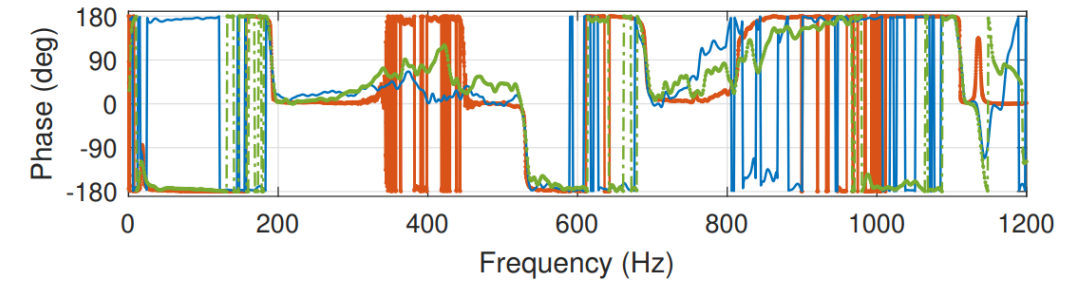
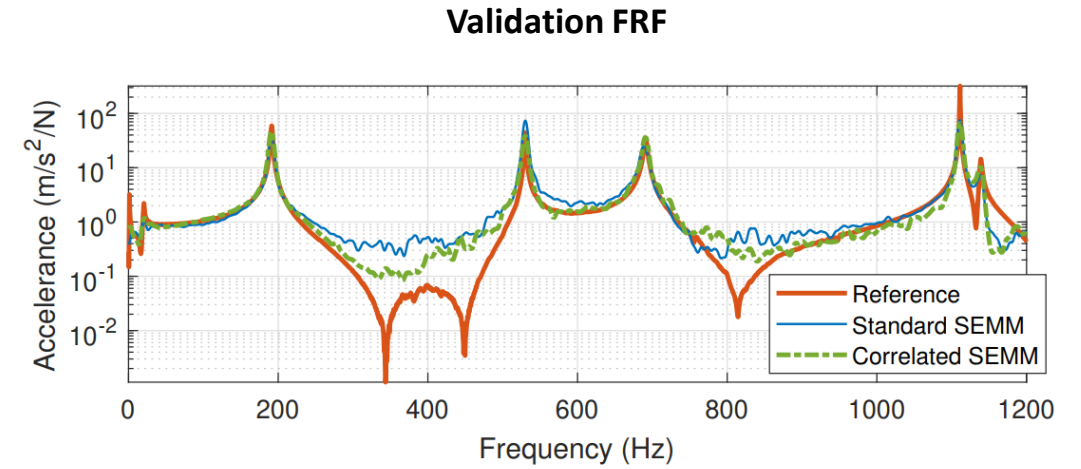
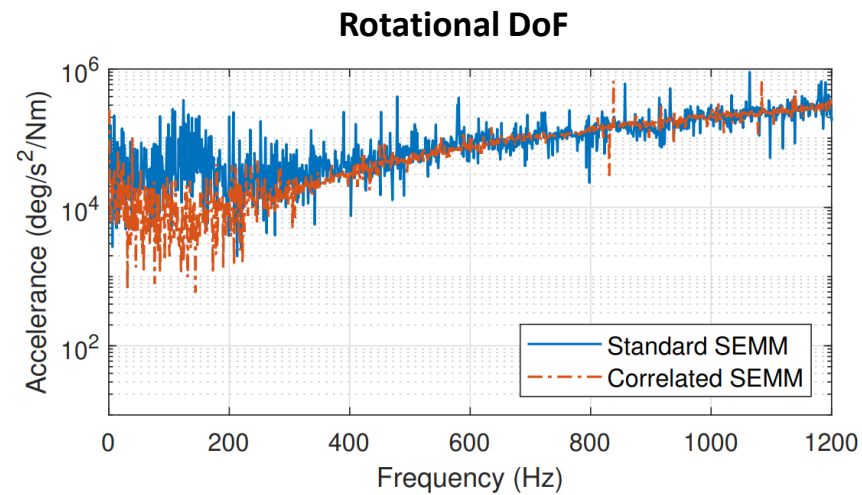
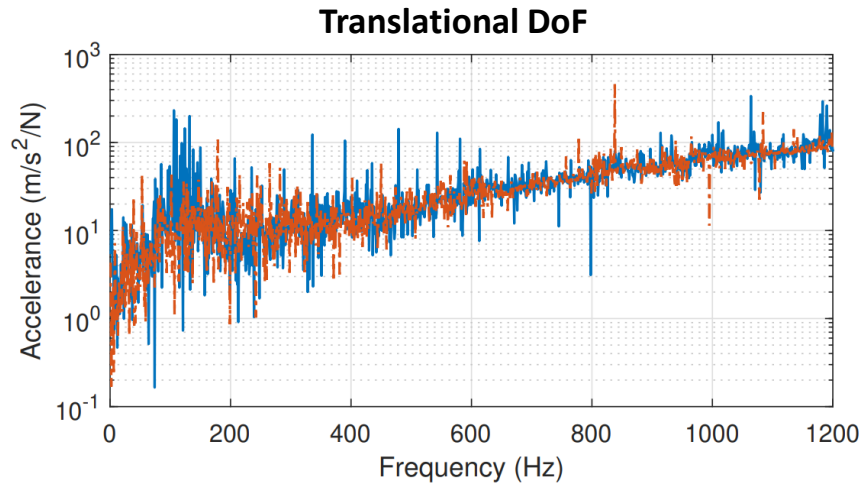
Removing correlated errors in the disk model



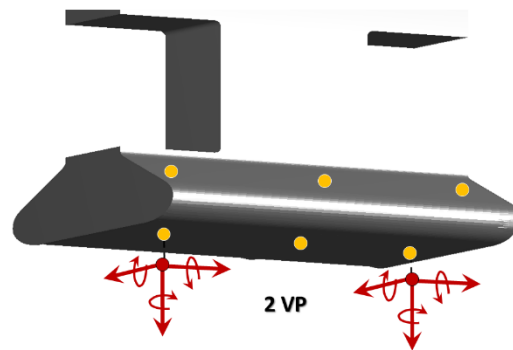
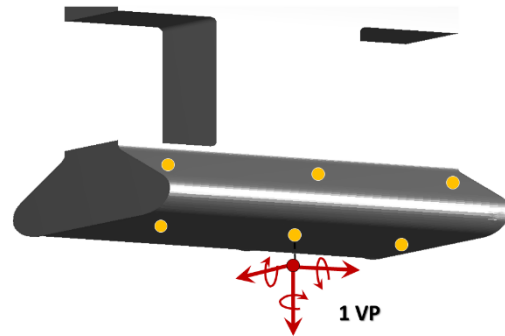
Disk tested in free condition



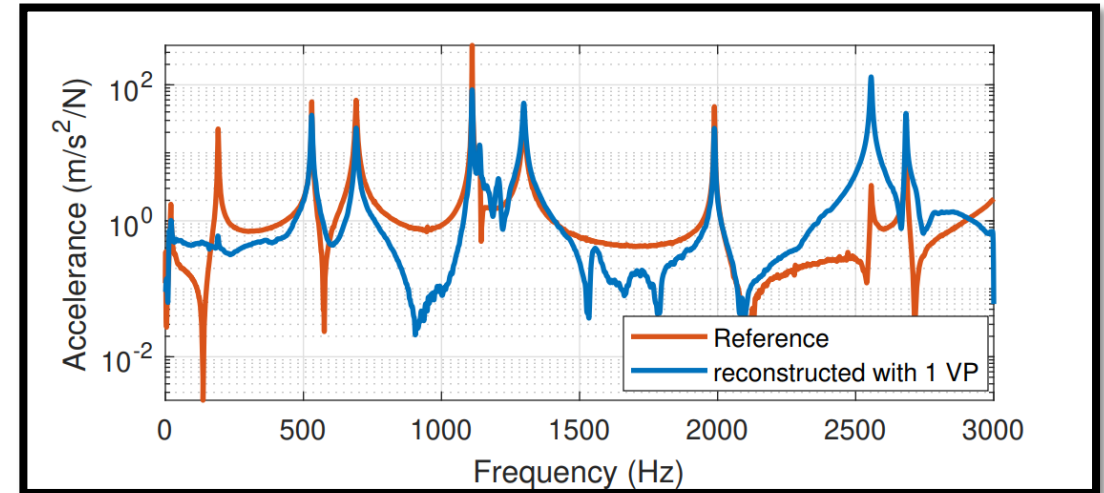
Comparing joint identification by standard and correlated SEMM



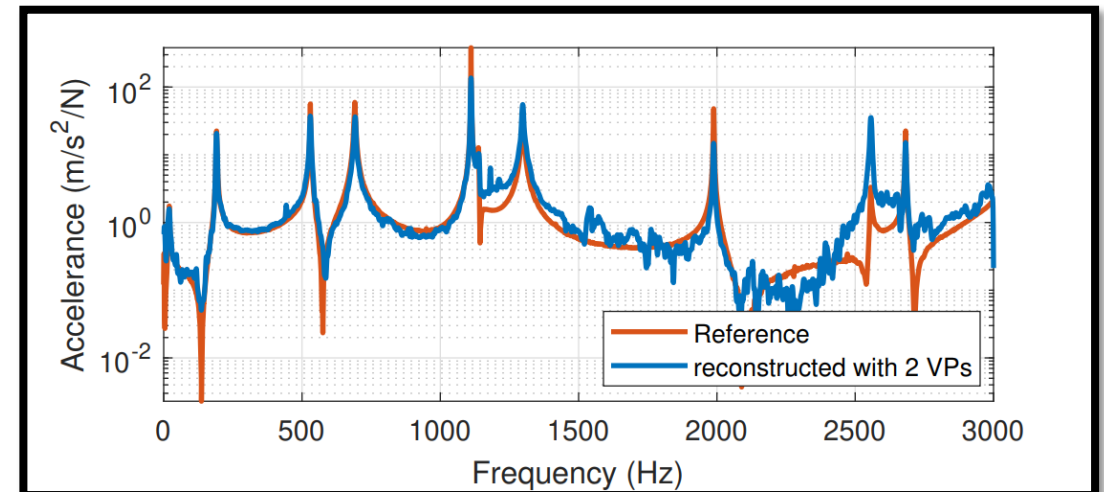
Other key parameters affecting the joint identification



1 VP



2 VP



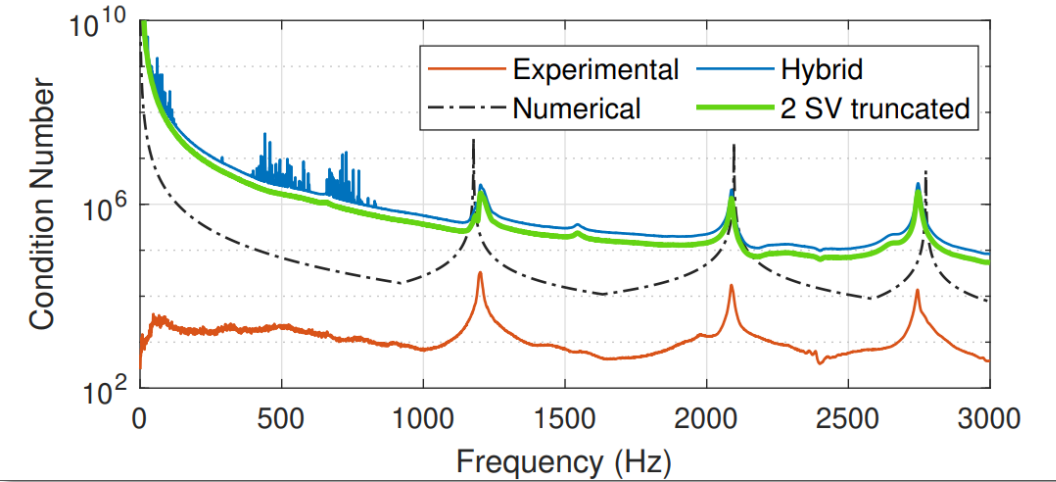
Other key parameters affecting the joint identification

$$\mathbf{Y}^S = \mathbf{Y}_{gg}^N - \mathbf{Y}_{gg}^N (\mathbf{Y}_{cg}^N)^+ (\mathbf{Y}_{ce}^N - \mathbf{Y}^{ov}) (\mathbf{Y}_{ge}^N)^+ \mathbf{Y}_{gg}^N$$

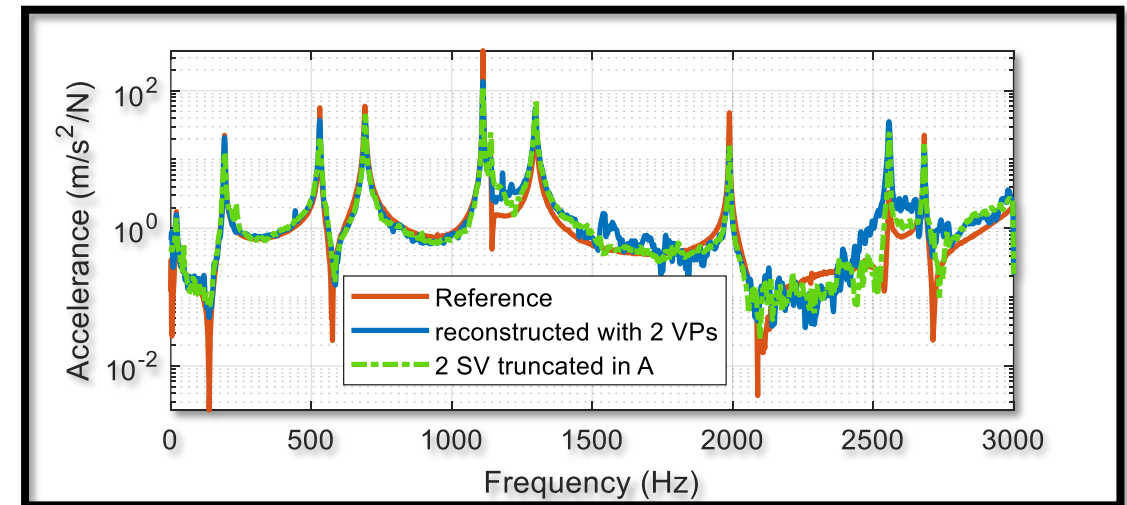
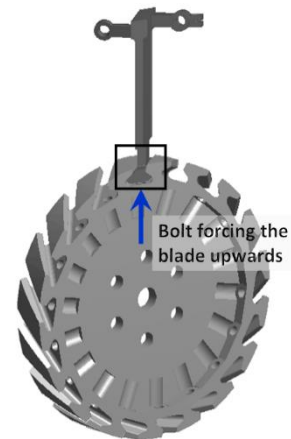
$$\mathbf{P} = \mathbf{U} \mathbf{S} \mathbf{V}^T$$

$$\mathbf{P}^+ = \mathbf{V} \mathbf{S}^{-1} \mathbf{U}^T$$

$$\mathbf{P}^+ = \sum_j^N \mathbf{v}_j \sigma_j \mathbf{u}_j^T \approx \sum_j^{k < N} \mathbf{v}_j \sigma_j \mathbf{u}_j^T$$



$$\mathbf{Y}^{AJB} = fbs(\hat{\mathbf{Y}}^{S,A}, \mathbf{Y}^J_{converged}, \mathbf{Y}^{S,B})$$

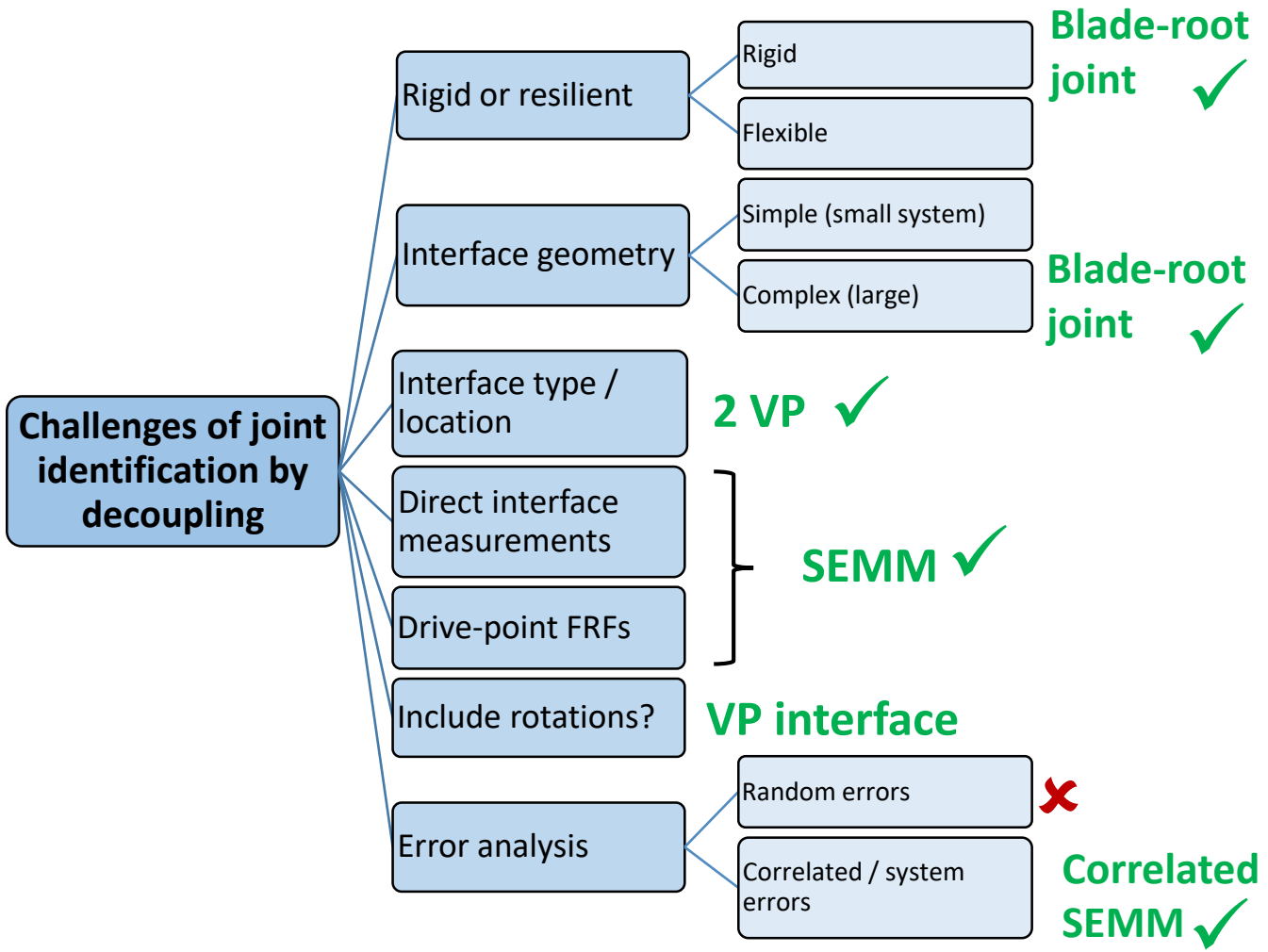


Conclusions

- Full experimental dynamic substructuring is extremely challenging even for beams
- Available approaches are hybrid yet limited
- For a blade-root type interface:
 - the joint is close to rigid
 - expansion by SEMM
- First application of a SEMM-based identification strategy on a blade-root
- Measurement and modelling errors affect the joint identification
- Weighted inverse calculations to subside errors in the sub-models
- Newly proposed correlated based approach (correlated SEMM)
 - reduces the errors in the identified joint
 - Better produces the validation FRF(s) around the resonances and closer to low response (anti-resonance) regions
- Joint system is better described by two virtual points (VP) than one VP
 - 12 DoF per component
- In the presence of noise, SVD filters for the submodels are also beneficial
- From a narrow band identification (~200Hz) → broad band (~3000 Hz)

Conclusions

Future work



- Robust fitting for fluctuating joint acceleration
- SEMM as an expansion method needs to be explored for:
 - Optimal expansion basis (numerical FRFs)
 - Optimal sensor locations
- Uncertainty quantification
- Extension to more than one joint

THANK YOU FOR YOUR ATTENTION!

Questions?

References

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List of own publications

<u>Ref.</u>	<u>Publication Details</u>	<u>Type</u>	<u>Status</u>
Saeed 2019 JPCS	Z. Saeed, C.M. Firrone, T.M. Berruti, Substructuring for Contact Parameters Identification in Bladed-disks, J. Phys. Conf. Ser. 1264 (2019) 012037. https://doi.org/10.1088/1742-6596/1264/1/012037 .	Conf. Paper	Published
Saeed 2020 JVA	Z. Saeed, S.W.B. Klaassen, C.M. Firrone, T.M. Berruti, D.J. Rixen, Experimental Joint Identification Using System Equivalent Model Mixing in a Bladed Disk, J. Vib. Acoust. 142 (2020). https://doi.org/10.1115/1.4047361	Journal Article	Published
Saeed 2020 ISMA	Z. Saeed, M. Kazeminasab, C.M. Firrone, T.M. Berruti, Improved identification of a blade-disk coupling through a parametric study of the dynamic hybrid models, in: ISMA 2020 - Int. Conf. Noise Vib. Eng., KU Leuven, Virtual Conference, 2020.	Conf. Paper	Accepted, Presented, To be published
Saeed 2020 TE	Z. Saeed, C.M. Firrone, T.M. Berruti, Hybrid Numerical-Experimental Model Update Based on Correlation Approach for Turbine Components, in: ASME Turbo Expo, Virtual Conference, 2020: pp. 1–12.	Conf. Paper	Accepted, Presented, To be published
Saeed 2020 JEGTP	Z. Saeed, C.M. Firrone, T.M. Berruti, Hybrid Numerical-Experimental Model Update Based on Correlation Approach for Turbine Components, ASME J. Eng. Gas Turbine Power. (2020) 1–22.	Journal Article	Accepted, To be published
Saeed 2021 JSV	Z. Saeed, C.M. Firrone, T.M. Berruti, Joint identification through hybrid models improved by correlations, J. Sound Vib. 494 (2021) 115889. https://doi.org/10.1016/j.jsv.2020.115889	Journal Article	Published
Saeed 2020 IMAC	Z. Saeed, G. Jenovencio, S. Arul, J. Blahoš, A. Sudhakar, L. Pesaresi, J. Yuan, F. El Haddad, H. Hetzler, L. Salles, A Test-Case on Continuation Methods for Bladed-Disk Vibration with Contact and Friction, in: G. Kerschen, M. Brake, L. Renson (Eds.), Nonlinear Struct. Syst. Vol. 1. Conf. Proc. Soc. Exp. Mech. Ser., Springer, Cham, 2020: pp. 209–212. https://doi.org/10.1007/978-3-030-12391-8_27 .	Book Chapter	Published
Saeed 2020 Eurodyn	G. Jenovencio, A. Sivasankar, Z. Saeed, D. Rixen, A DELAYED FREQUENCY PRECONDITIONER APPROACH FOR SPEEDING-UP FREQUENCY RESPONSE COMPUTATION OF STRUCTURAL COMPONENTS, in: EURO DYN 2020 XI Int. Conf. Struct. Dyn., Virtual Conference, Athens, 2020: pp. 56–66.	Conf. Paper	Accepted, Presented, To be published