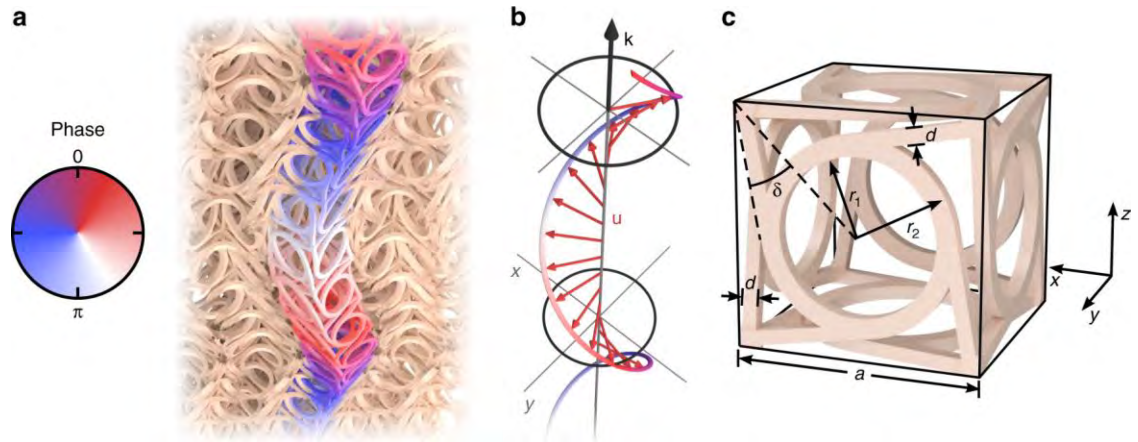
PhD in Mechanical Engineering

Development of novel design methodologies for additive manufacturing



- **Candidate**: Ing. Riccardo Caivano
- **Tutor**: Prof. Giorgio Chiandussi
- **Scholar grant**: Interdepartmental Integrated Additive Manufacturing@PoliTo Centre (IAM) - Design Additive Manufacturing

Frenzel. *et al. Nat Commun* **10,** 3384 (2019)

Topology Optimisation for 3D printed composite materials

Problem: Finding the best material distribution and fibre orientation for a composite material

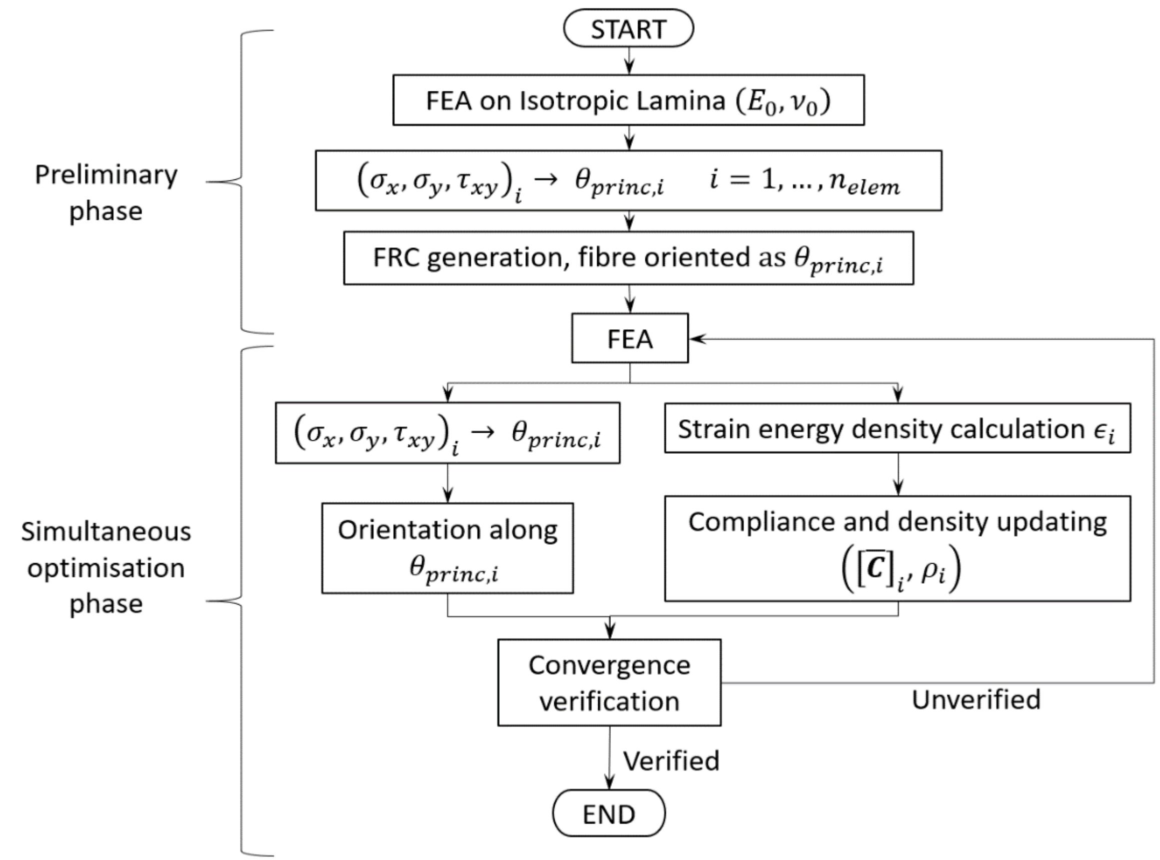
State of art: Many algorithms but computationally expensive and applied on bidimensional cases only

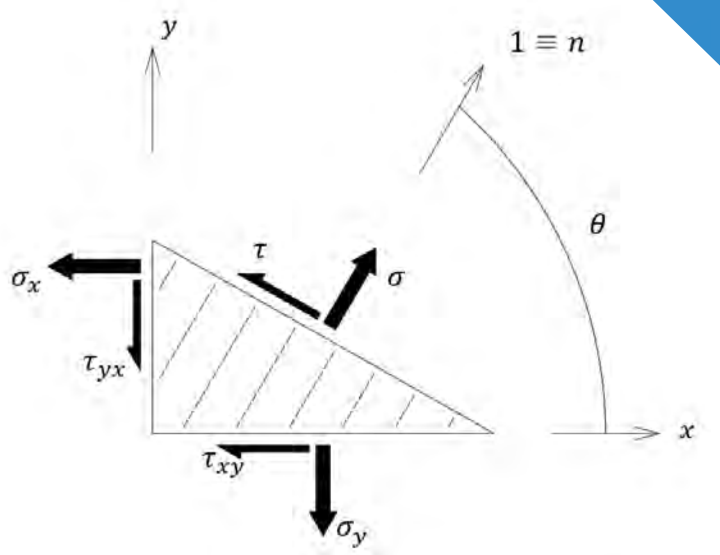
Proposed solution: novel algorithm based on optimality criteria for topology optimisation and tailored fibre orientation suitable for 3D printed composites

Significance: Novel generation of strong, light and fully optimised composite components

R. Caivano, A. Tridello, D. Paolino, G. Chiandussi, *Topology and fibre orientation simultaneous optimisation: A design methodology for fibre-reinforced composite components*. Proceedings of the Institution of Mechanical Engineers, Part L: Journal of Materials: Design and Applications, 234(9) (2020) 1267–1279. https://doi.org/10.1177/1464420720934142

Proposed algorithm solution





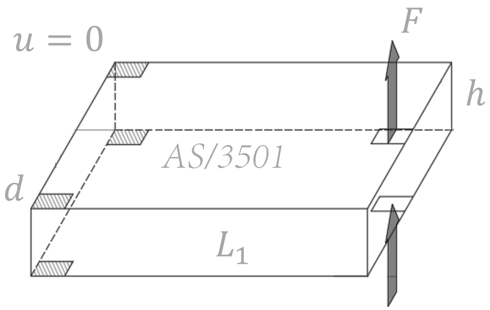
Orientation of fibre reinforcement along principal axes in each element

$$[\bar{C}]_i^{j+1} = \frac{\bar{\epsilon}^j}{\epsilon_i^j}[\bar{C}]_i^j \rightarrow \epsilon_i = \frac{\frac{1}{2}\boldsymbol{e_i}^T(\boldsymbol{u})[\boldsymbol{A}]\boldsymbol{e_i}(\boldsymbol{u})\Omega_i}{\Omega_i}$$

Updating law based on optimality criteria:
Uniform strain energy density

Legend

FEA - Finite Element Analysis
E_0, ν_0 - Stiffness and necking coefficient
$(\sigma_x, \sigma_y, \tau_{xy})_i$ - element vector stress
$\theta_{princ,i}$ - element principal stress angle
$[\boldsymbol{A}]$ - stiffness matrix
$\boldsymbol{e}_i$ - element strain vector
j - iteration number
i - element number
FRC - Fibre Reinforced Composite
Ω_i - element volume
ϵ_i - strain energy density
$[\bar{\boldsymbol{C}}]_i$ - element rotated compliance matrix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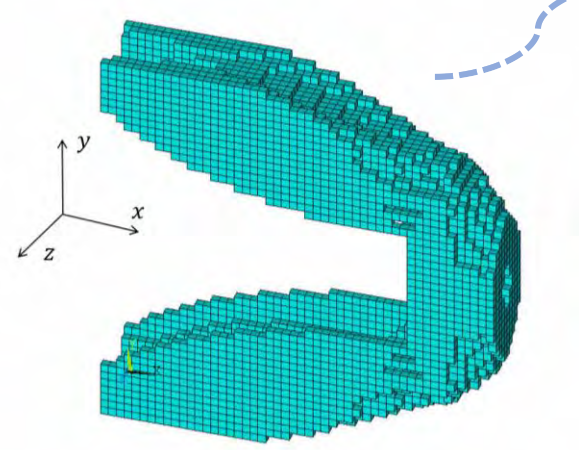
F
h
AS/3501
d
L1


y
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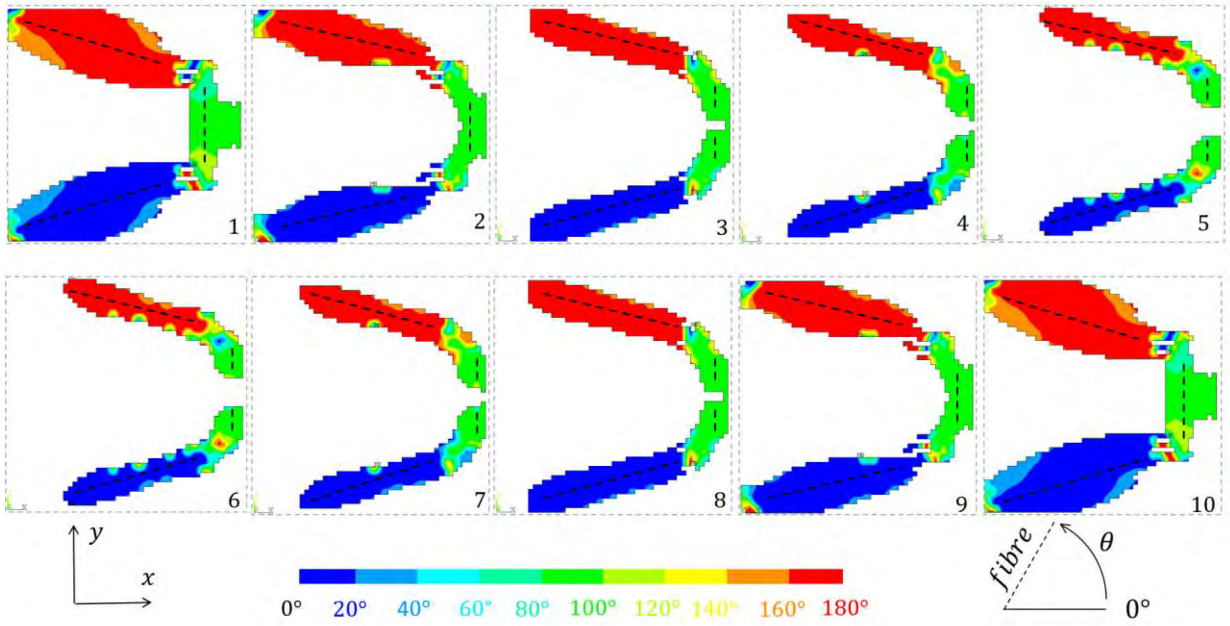
1
2
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6
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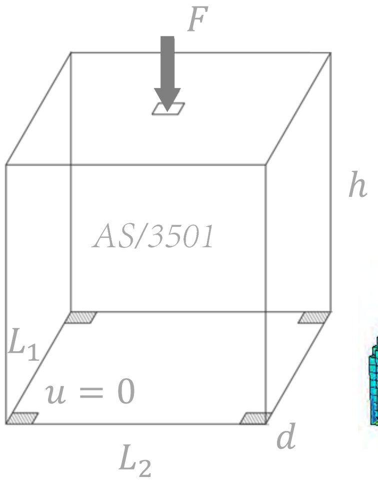
10
y
x
0° 20° 40° 60° 80° 100° 120° 140° 160° 180°
fibre
θ
0°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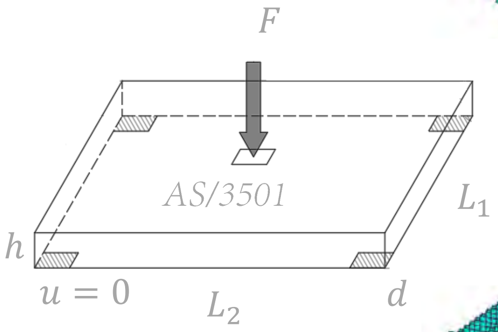
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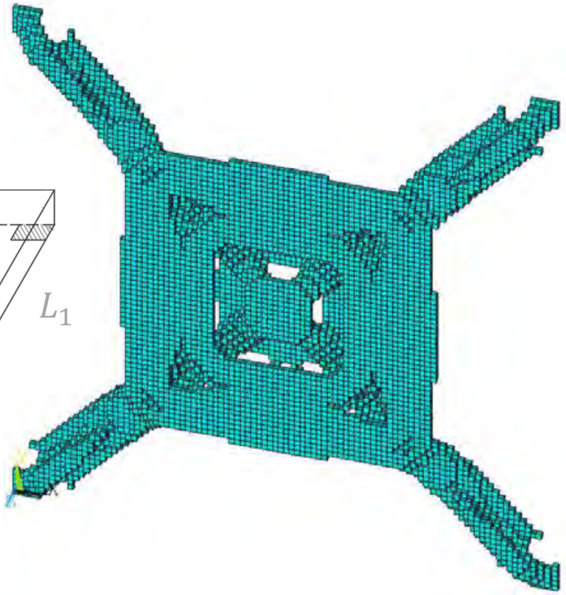


Thermo-structural Topology Optimisation

Problem: Finding the best material distribution to minimise structural compliance and maximise heat exchange concurrently

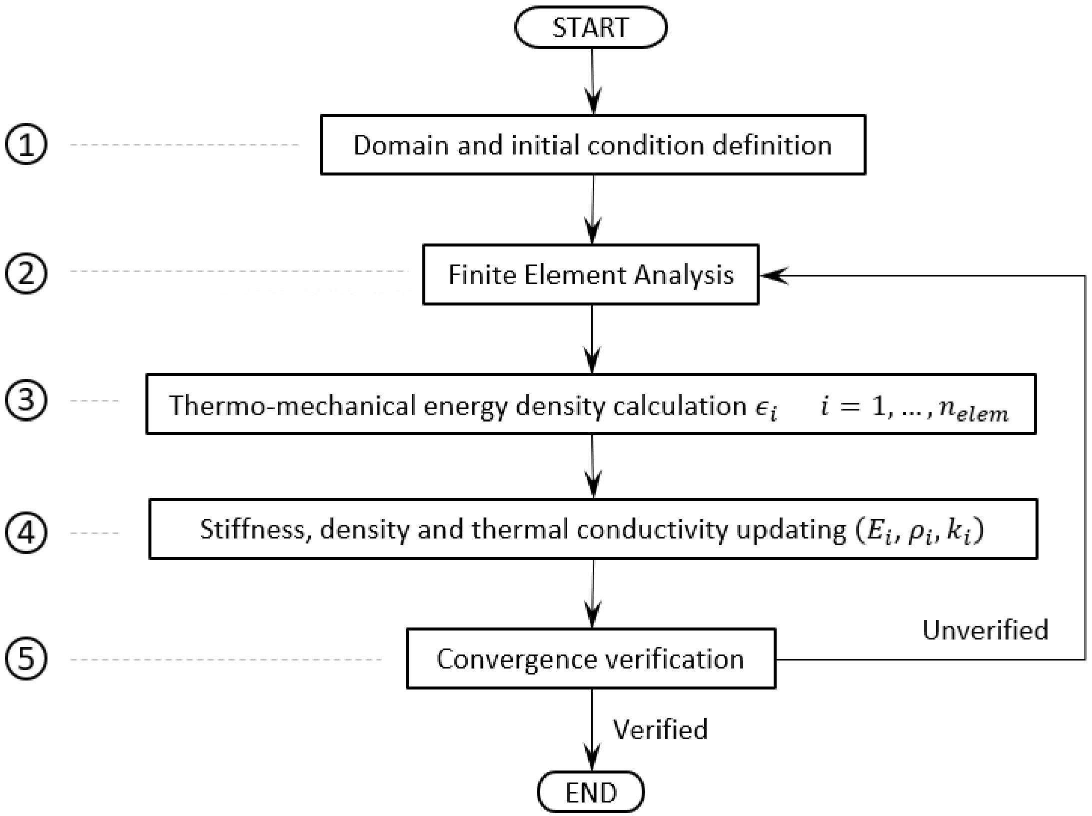
State of art: Few attempts in thermo-elastic topology optimisation, no coupled constraints

Proposed solution: novel algorithm based on optimality criteria for thermo-structural topology optimisation

Significance: the final optimised component guarantees the maximum stiffness and concurrently the maximum heat exchange, improving lightness and weight saving

R. Caivano, A. Tridello, M. Codegone, G. Chiandussi, *A new methodology for thermo-structural topology optimization: analytical definition and validation*. (Peer Review in progress)

Proposed algorithm solution



Key definition:

Thermo-structural potential energy

$$\Pi(\boldsymbol{u}, \theta) = -\frac{1}{2}\left(\int_{\Omega} \boldsymbol{e}^T(\boldsymbol{u})\boldsymbol{A}\boldsymbol{e}(\boldsymbol{u})\, d\boldsymbol{x} + \xi \int_{\Omega} k\nabla^2\theta\, d\boldsymbol{x} + \int_{\Omega} \beta\, \nabla^T\theta\, \boldsymbol{u}\, d\boldsymbol{x}\right)$$

Updating law based on optimality criteria:

Uniform thermo-structural energy density

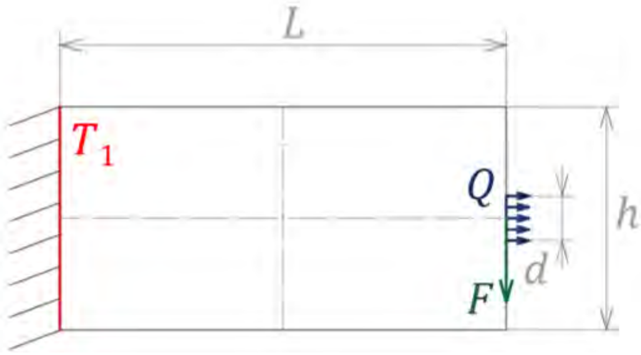
$$\epsilon_i = \frac{\frac{1}{2}\left(\boldsymbol{e_i}^T(\boldsymbol{u})\boldsymbol{A}\boldsymbol{e_i}(\boldsymbol{u}) + \xi\nabla_i^T\theta k(\boldsymbol{x})\nabla_i\theta + \beta\, \nabla_i^T\theta\, \boldsymbol{u_i}\right)\Omega_i}{\Omega_i}$$

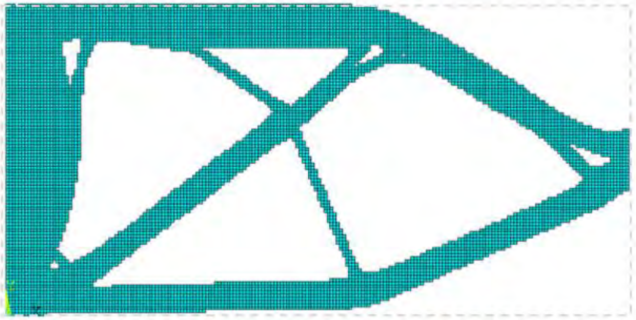
$$\begin{cases} E_i^{j+1} = \dfrac{\epsilon_i^j}{\epsilon^j} E_i^j \\ \rho_i^{j+1} = \dfrac{\epsilon_i^j}{\epsilon^j} \rho_i^j \\ k_i^{j+1} = \dfrac{\epsilon_i^j}{\epsilon^j} k_i^j \end{cases}$$

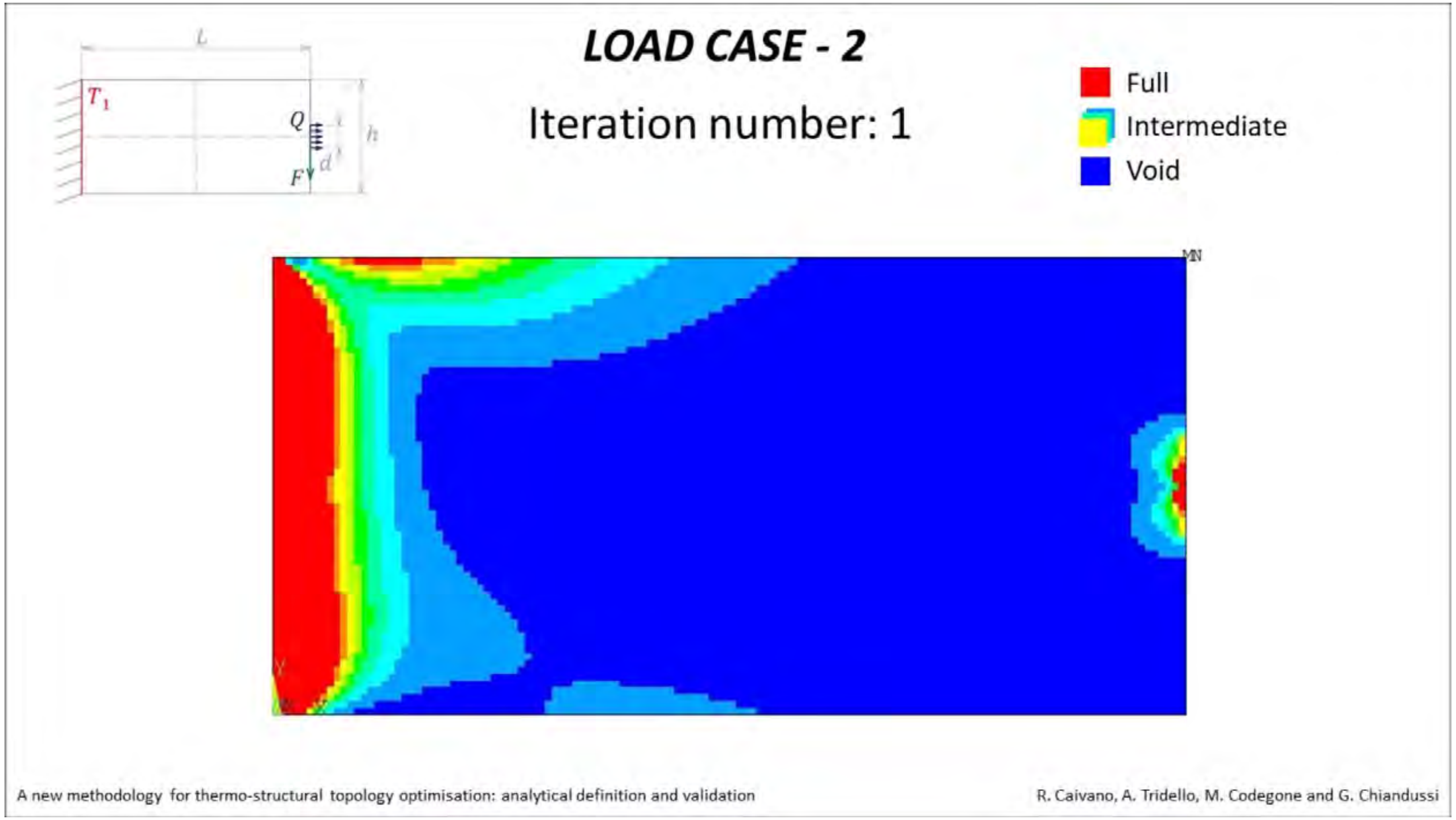
Legend

$\Pi(\boldsymbol{u}, \theta)$ - thermo-structural potential energy
$\theta_{princ,i}$ - element principal stress angle
$[\boldsymbol{A}]$ - stiffness matrix
$\boldsymbol{e_i}$ - element strain vector
ξ - energy linker term
E_i - element stiffness module
j - iteration number
i - element number
Ω_i - element volume
k_i - element thermal conductivity
ρ_i - element mass density
ϵ_i - element thermo-structural energy density

Application and test case example







Elasto-static cloaking

Problem: Finding the material distribution which is able to cloak a hole or material variation within a component

State of art: Few attempts with lattice structures and pentamode materials only

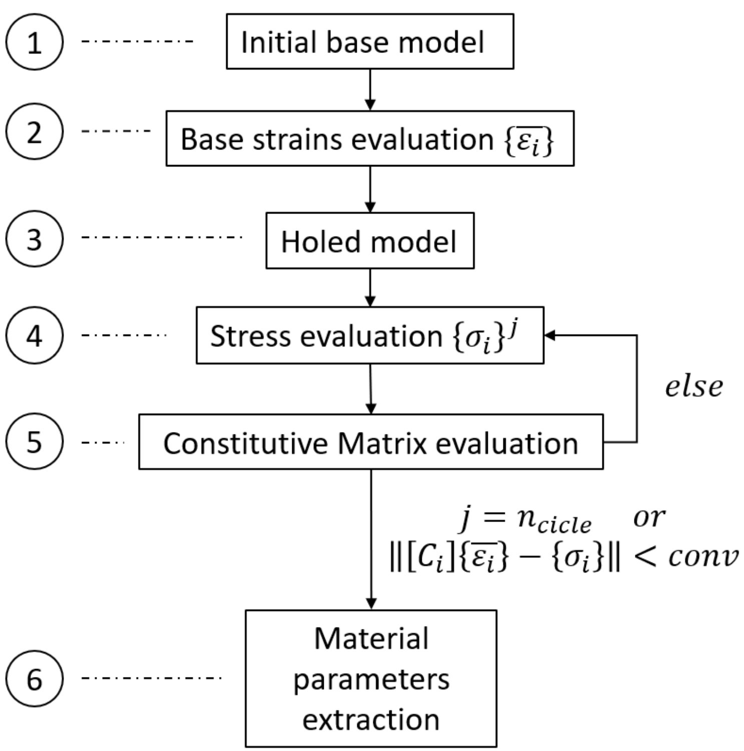
Proposed solution: novel algorithm based on numerical optimisation method for isotropic and anisotropic elastostatic cloaking

Significance: the provided material distribution overrides the effect of a hole, what is inside the hole is not affected by external forces, i.e. the perfect case.

R. Caivano, SS. Injeti, C. Dario,
A novel numerical approach for elastostatic cloaking

In collaboration with Daraio research group at Caltech

Proposed algorithm solution



Legend

$eig[C_i]$- eigenvalues of $[C_i]$
i - element number
j - iteration number
$[C_{i,min}]$- inferior bound
$[C_{i,MAX}]$- upper bound
$[\tilde{C}_e]$- filtered compliance
Δ - cloaking error
$conv$ - convergence parameter

$[C_i]$ - element compliance matrix
$\{\overline{\varepsilon_i}\}$ - reference element strain vector
$\{\sigma_i\}$ - element stress vector
$[K_i]$ - global stiffness matrix
$\{U_i\}$ - global displacement vector
$\{F_i\}$ - global force vector
Γ_e - domain external boundaries
$\boldsymbol{u}_i$- element displacement vector
$\boldsymbol{u}_i^0$- reference for $\boldsymbol{u}_i$

Optimisation problem formulation
Resolution by *fmincon* and *CVX* in MATLAB

$$\begin{cases} Find\ [C_i] \\ \min\limits_{[C_i]} \|[C_i]\{\overline{\varepsilon_i}\} - \{\sigma_i\}\| \\ s.t. \begin{cases} [K_i]\{U_i\} = \{F_i\} \\ \forall\ eig[C_i] > 0 \\ [C_i]\ symmetric \\ [C_{i,min}] \leq [C_{i,j}] \leq [C_{i,MAX}] \\ i = 1, \dots, n_{elem} \end{cases} \end{cases}$$

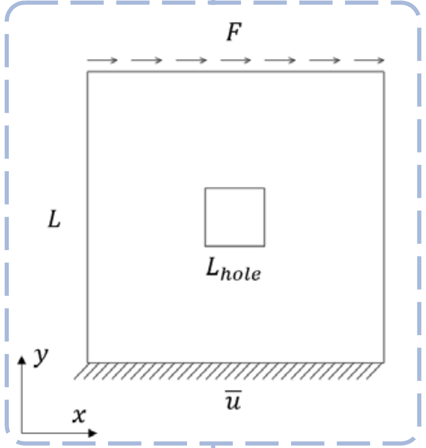
Constant filter application

$$[\tilde{C}_e] = \frac{\sum_{i\in\Omega_e}[C_i]}{\sum_{i\in\Omega_e} i}$$

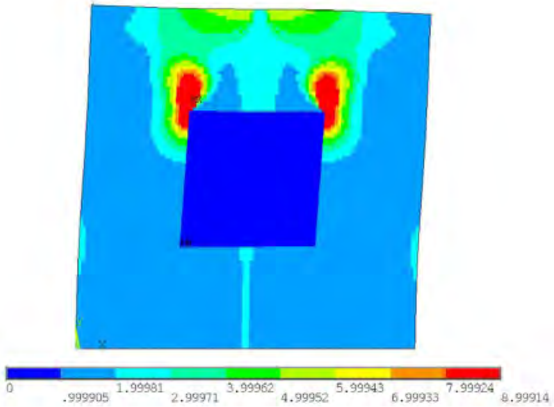
Cloaking effectiveness measure

$$\Delta = \frac{\sqrt{\sum_{i\in\Gamma_e}\left(\boldsymbol{u}_i - \boldsymbol{u}_i^0\right)^2}}{\sqrt{\sum_{i\in\Gamma_e}\left(\boldsymbol{u}_i^0\right)^2}}$$

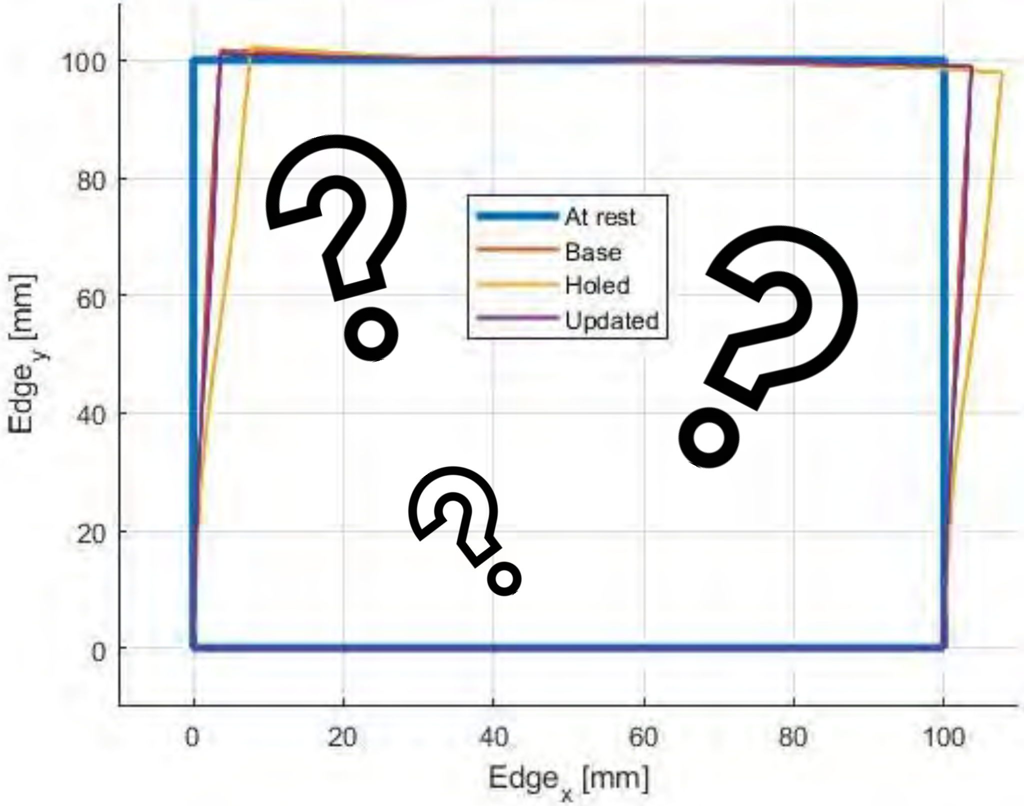
$\Delta \cong 5\%$



Isotropic material

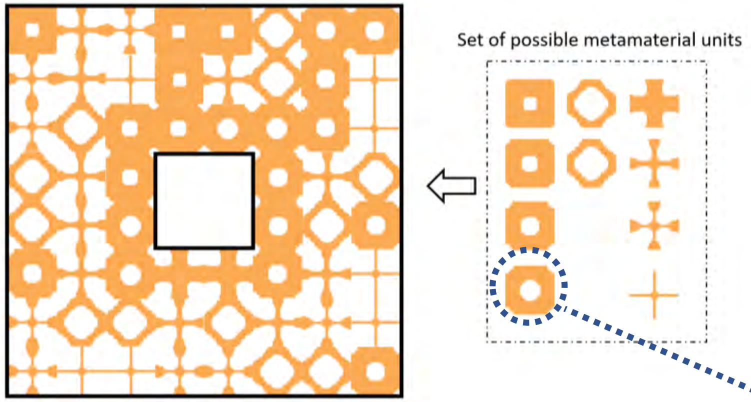


Is there any hole?



Anisotropic material and Metamaterial application

$\Delta \cong 1{,}7\%$



$$[C_i] = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ \cdots & C_{22} & C_{23} \\ Sym & \cdots & C_{33} \end{bmatrix}$$

Defect Driven Topology Optimisation

Problem: Finding the best material distribution to minimise structural compliance with stress constraint and <u>fatigue defect driven constraint</u>

State of art: <u>No previous research</u> focused on defect presence in topology optimisation, fatigue considered with classical theory only

Proposed solution: novel algorithm based on numerical optimisation for topology optimisation <u>including the presence of defects</u>

Significance: the final optimised component avoid static failure and fatigue failure thanks to the <u>non-propagability</u> of the most critical defect

R. Caivano, A. Tridello, D. Paolino, G. Chiandussi, F. Berto, X. Gao, Haitao Ma,
Minimum compliance topology optimization of continuum structures with defect driven constraints

In collaboration with NTNU and Guangdong University of Technology

NTNU
Norwegian University of Science and Technology

Topology optimisation formulation

$$find\ \boldsymbol{\rho} = \{\rho_1, \rho_2, \dots, \rho_{N_e}\}$$

$$\min_{\boldsymbol{\rho}} C = \boldsymbol{F}^T \boldsymbol{U} = \boldsymbol{U}^T \boldsymbol{K} \boldsymbol{U}$$

$$s.t. \begin{cases} \boldsymbol{K}\boldsymbol{U} = \boldsymbol{F} \\ V(\boldsymbol{\rho}) \le \bar{V} \\ \sigma_e^{\mathrm{vM}}(\boldsymbol{\rho}) \le \bar{\sigma}^s \\ \sigma_e^1(\boldsymbol{\rho}) \le \bar{\sigma}^f \quad if\ \sigma_1(\boldsymbol{\rho}) \ge 0 \\ \underline{\rho} \le \rho_e \le 1 \quad e = 1,2,\cdots,N_e \end{cases}$$

- Density based topology optimisation, SIMP approach
- SIMP issues → continuation method, linear filer, non-linear projection
- Stress constraints → *pq*-relaxation, K-S aggregation and STM based correction
- MMA solver for the numerical optimisation

Murakami fatigue approach

$$\bar{\sigma}^f = \frac{C_1 \cdot (HV + 120)}{\left(\mu_{\sqrt{a}} + \sigma_{\sqrt{a}} \cdot (-ln(-ln(F)) + ln(V/V_0))\right)^{\frac{1}{6}}} \cdot \left(\frac{1-R}{2}\right)^{0.226 + HV \cdot 10^{-4}}$$

- Defect population lower the fatigue strength
- LEVD defect distribution is supposed
- The most critical defect must not propagate
- 1st principal stress defect-driven constraint

Legend

$\boldsymbol{\rho}$ - density variable vector
C - compliance
$\boldsymbol{K}$ - global stiffness matrix
$\boldsymbol{U}$ - global displacement vector
$\boldsymbol{F}$ - global force vector
N_e - number of elements
R - stress ratio

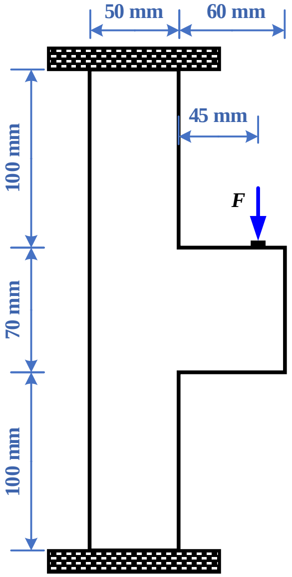
V - volume
$\bar{V}$ - volume constraint
σ_e^{vM} - element von mises stress
$\bar{\sigma}^s$ - max admissible vM stress
σ_e^1 - element 1st principal stress

C_1 - defect location parameter
HV - Vickers hardness
$\mu_{\sqrt{a}}$ - location LEVD parameter
$\sigma_{\sqrt{a}}$ - scale LEVD parameter
F – probability
V_0 - reference volume

Application and test case example: Corbel structure with double re-entrant corner

Imposed limits:

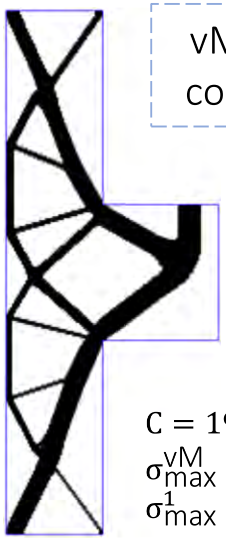
$$\begin{cases} \bar{\sigma}^{s} = 660\ MPa \\ \bar{\sigma}^{f} = 505\ MPa \end{cases}$$



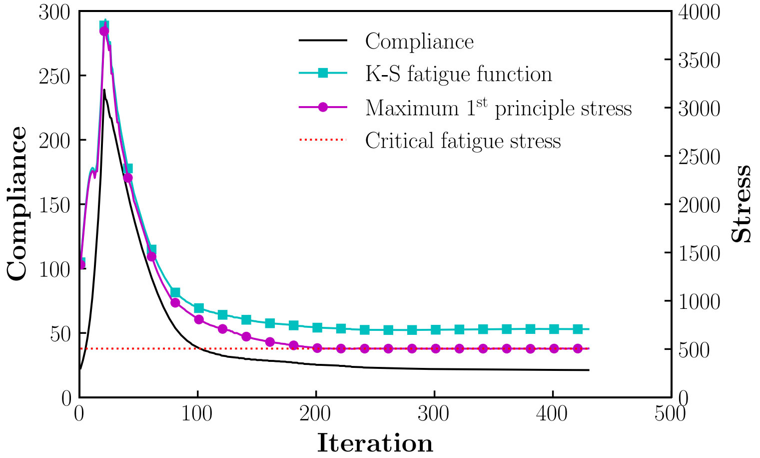
No stress constraint

C = 16.96 Nmm
σ^{vM}_{max} = 1309.00 MPa
σ^{1}_{max} = 1435.95 MPa

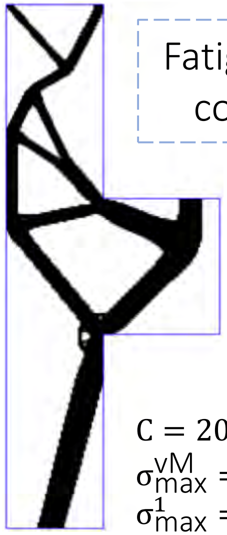
vM stress constraint



C = 19.79 Nmm
σ^{vM}_{max} = 659.75 MPa
σ^{1}_{max} = 666.80 MPa

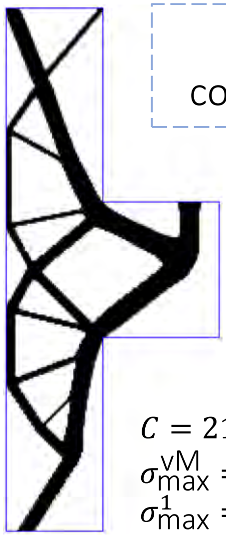


Fatigue stress constraint



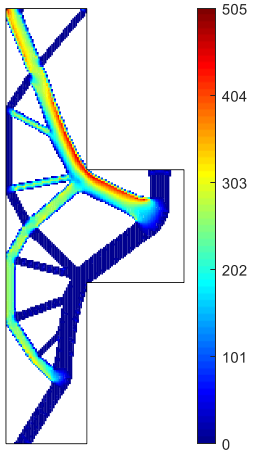
C = 20.98 Nmm
σ^{vM}_{max} = 1753.96 MPa
σ^{1}_{max} = 504.98 MPa

All constraints

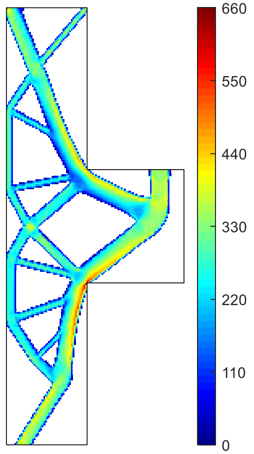


C = 21.33 Nmm
σ^{vM}_{max} = 659.01 MPa
σ^{1}_{max} = 504.97 MPa

1st principal stress [MPa]

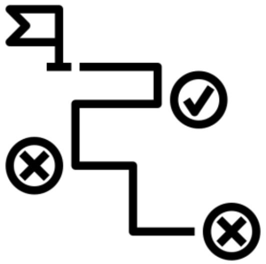


Von Mises stress [MPa]



Future outlook for 3rd year PhD

- Defect driven topology optimisation:
  - History load case
  - Complex defect population
  - Cracked initial domains



- Metamaterial geometry extraction from generic compliance matrix

- Dynamic Topology Optimisation on 3D composite