

# Advanced and nonlocal theories for the multi-scale/multi-field analysis of structures



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Joint research project with top university founded by Compagnia di San Paolo  
(City University of Hong Kong)



## Methodology

Carrera Unified Formulation allows the three-dimensional displacement field  $u(x, y, z)$  to be expressed as a general expansion of the primary unknowns. In the case of one-dimensional theories, one has:

$$u(x, y, z) = F_s(x, z)u_s(y), \quad s = 1, 2, \dots, M$$

$F_s$ : functions of the coordinate  $x$  and  $z$  on the cross-section  
 $u_s$ : vector of the generalized displacements, along the beam axis  
 $M$ : number of the terms used in the expansion.

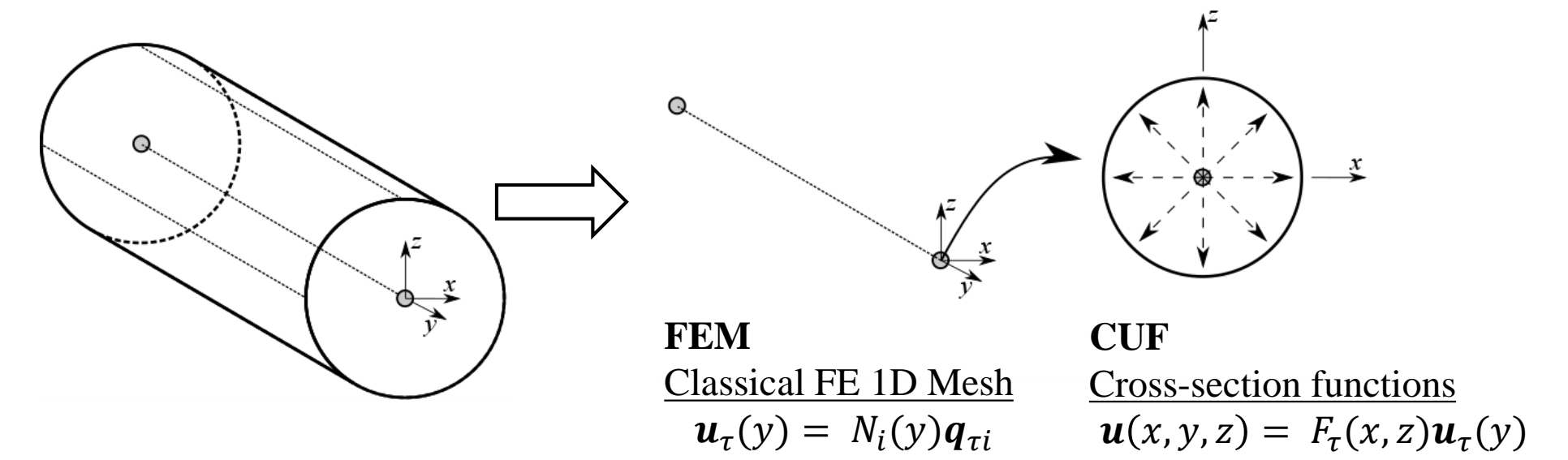
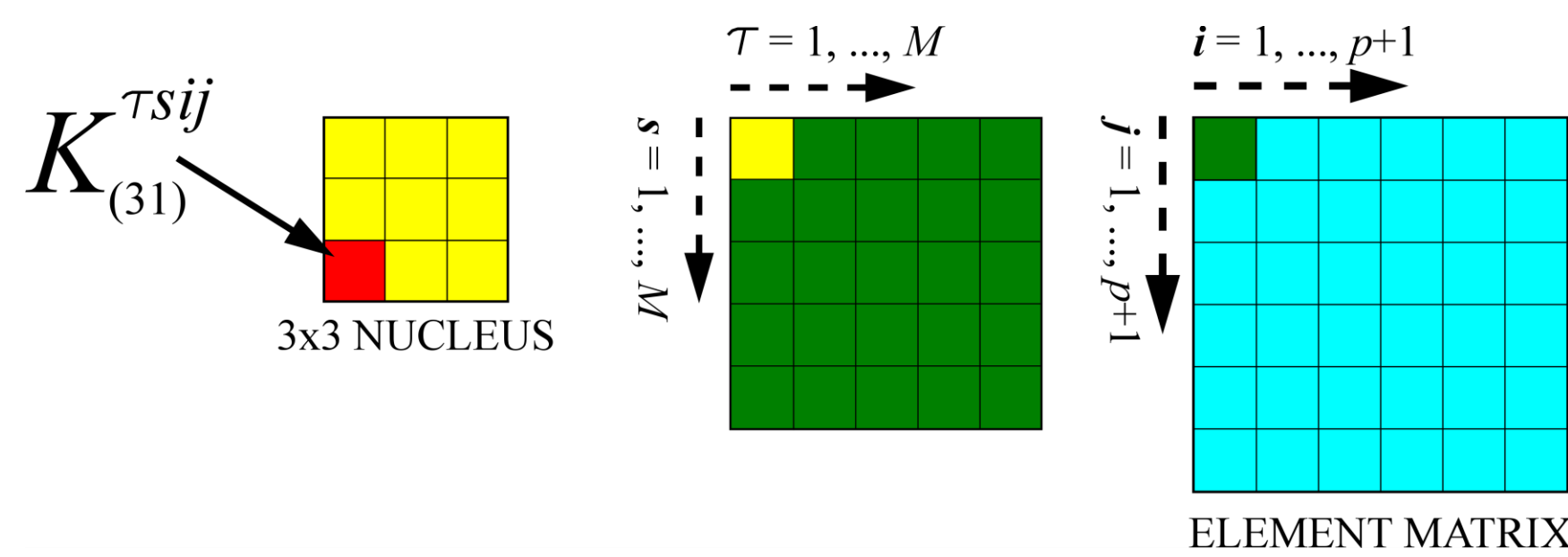
From the principle of virtual work:

$$\delta L_{\text{int}} = \delta \mathbf{q}_{\tau i}^T \mathbf{K}^{ijrs} \mathbf{q}_{sj}$$

$$\delta L_{\text{ine}} = \delta \mathbf{q}_{\tau i}^T \mathbf{M}^{ijrs} \ddot{\mathbf{q}}_{sj}$$

$$\delta L_{\text{ext}} = \delta \mathbf{q}_{\tau i}^T \mathbf{P}^{ir}$$

Fundamental nuclei



$$\text{CUF + FEM: } u(x, y, z) = F_\tau(x, z)N_i(y)q_{\tau i}$$

## Geometrical Nonlinearity

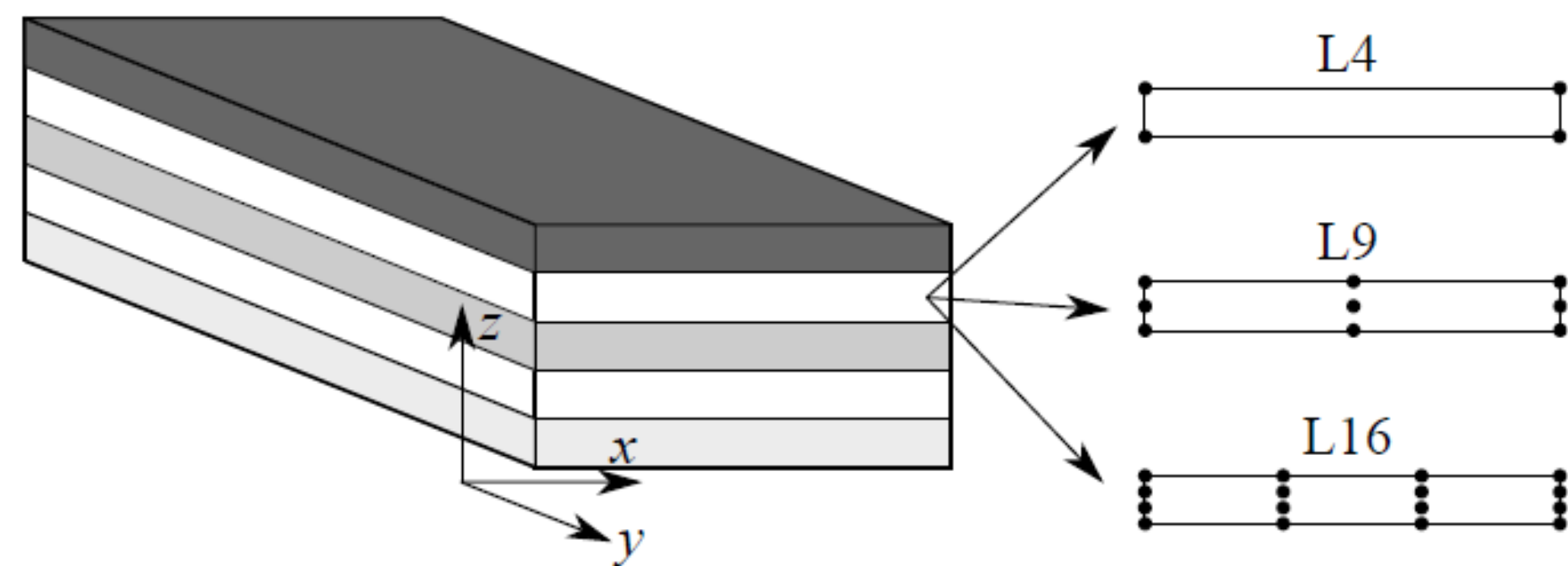
### Formulation

$$\varepsilon = \varepsilon_l + \varepsilon_{nl} = (\mathbf{b}_l + \mathbf{b}_{nl})\mathbf{u} \rightarrow \{u_x \ u_y \ u_z\}^T$$

$$\begin{bmatrix} \partial_x & 0 & 0 \\ 0 & \partial_y & 0 \\ 0 & 0 & \partial_z \\ \partial_z & 0 & \partial_x \\ 0 & \partial_z & \partial_y \\ \partial_y & \partial_x & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{2}(\partial_x)^2 & \frac{1}{2}(\partial_x)^2 & \frac{1}{2}(\partial_x)^2 \\ \frac{1}{2}(\partial_y)^2 & \frac{1}{2}(\partial_y)^2 & \frac{1}{2}(\partial_y)^2 \\ \frac{1}{2}(\partial_z)^2 & \frac{1}{2}(\partial_z)^2 & \frac{1}{2}(\partial_z)^2 \\ \partial_x \partial_z & \partial_x \partial_z & \partial_x \partial_z \\ \partial_y \partial_z & \partial_y \partial_z & \partial_y \partial_z \\ \partial_x \partial_y & \partial_x \partial_y & \partial_x \partial_y \end{bmatrix}$$

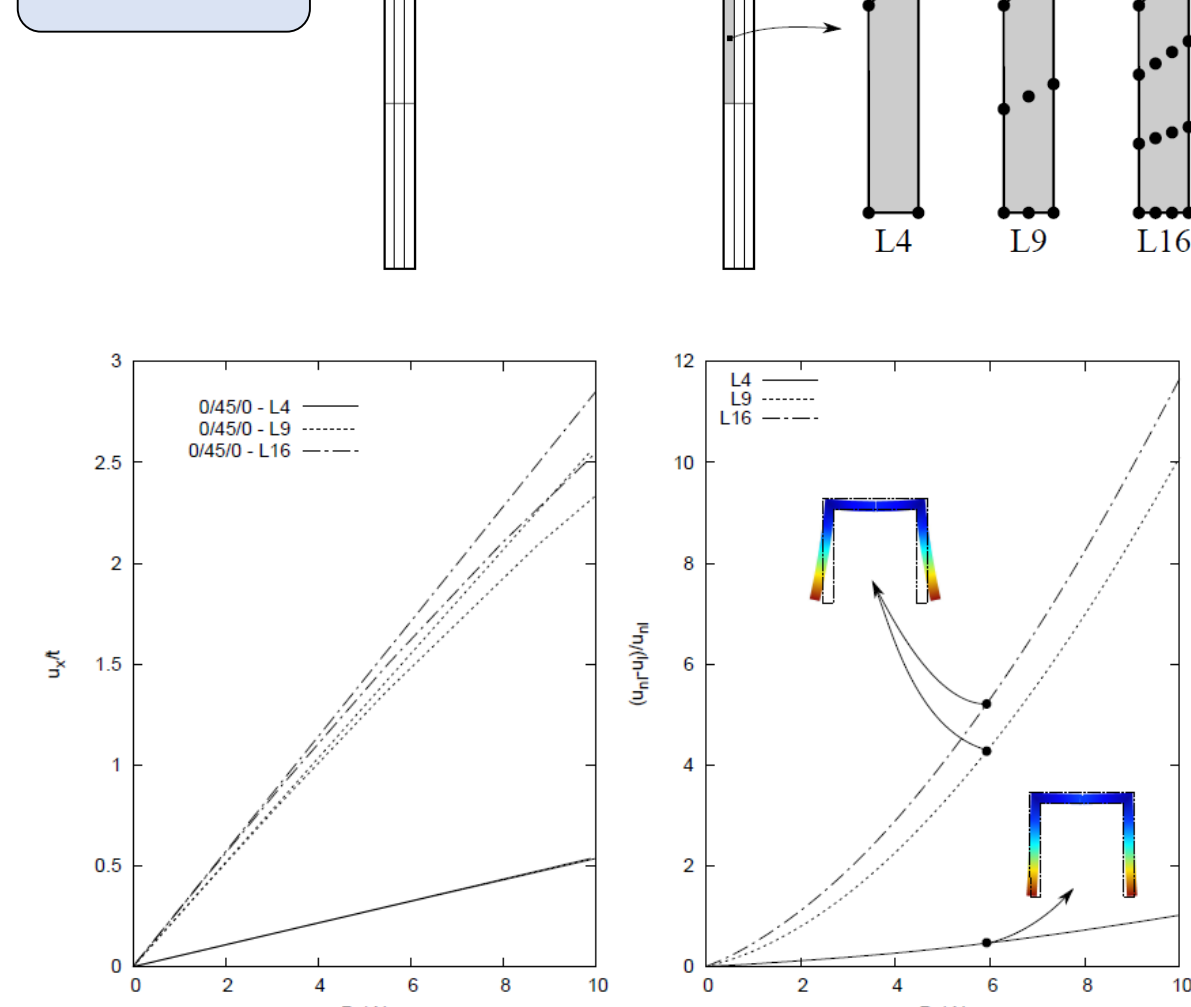
### Layerwise approach

- Layerwise approximation, by employing a Lagrange polynomial discretization per layer.
- Every layer has its own kinematics described.

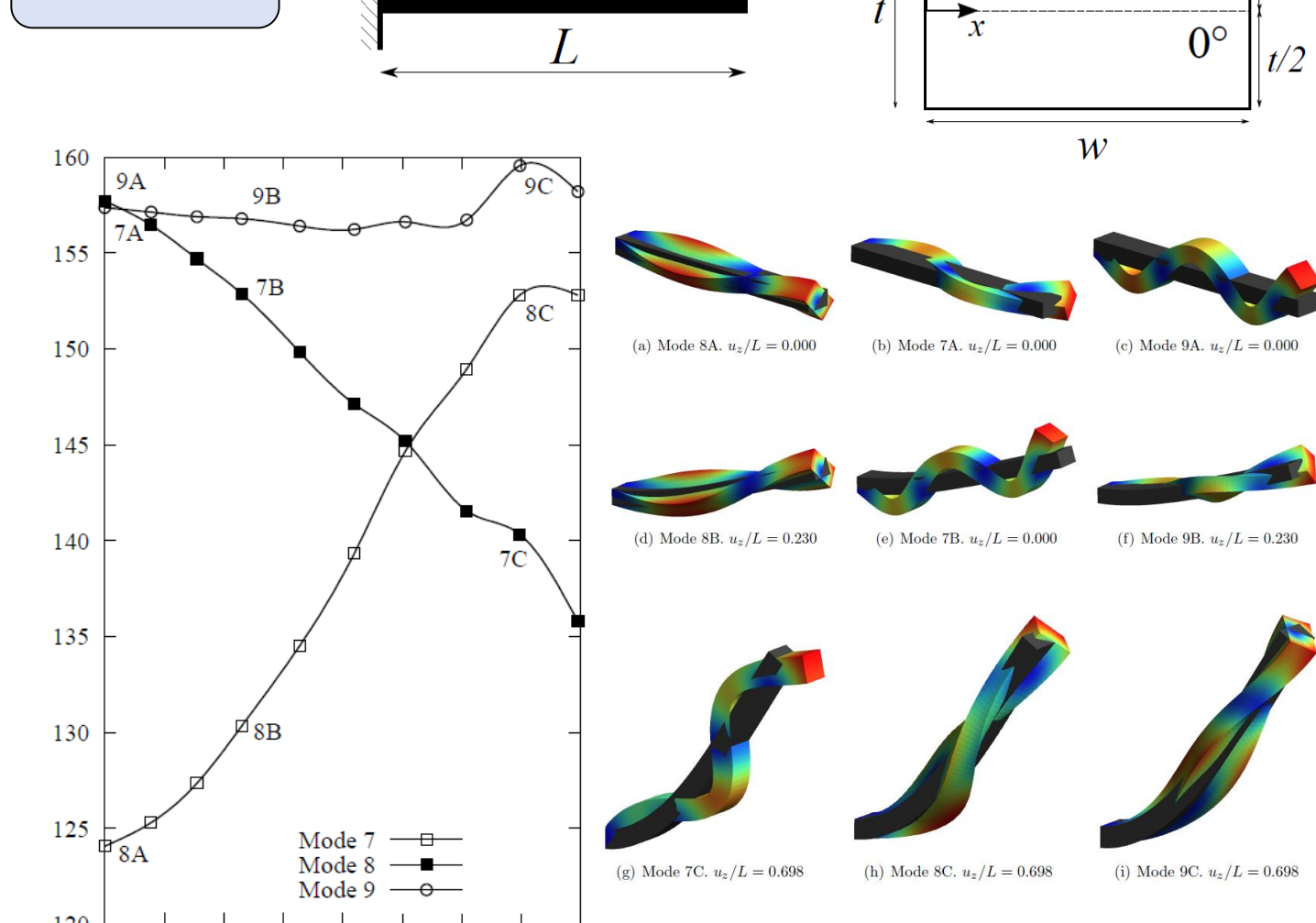


### Results

#### Static



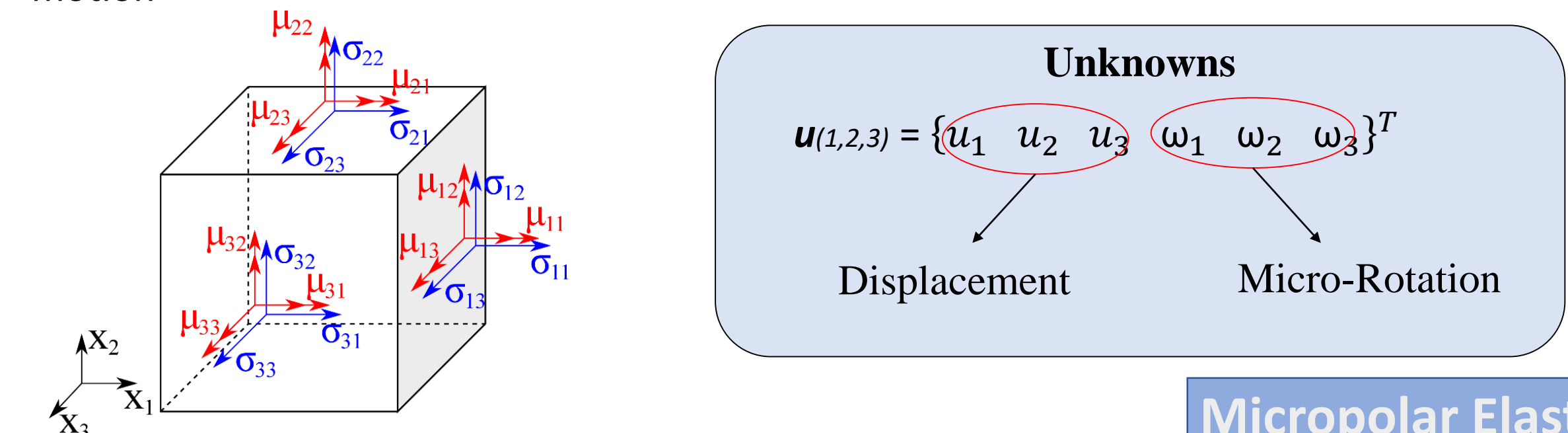
#### Dynamic



## Micropolar Elasticity

### Nodal unknowns

In Micropolar Elasticity, every particle of the material is meant to be able to rotate indepently of the surrounding particles, and so additional unknown rotations are needed to describe the motion



### Micropolar Elasticity

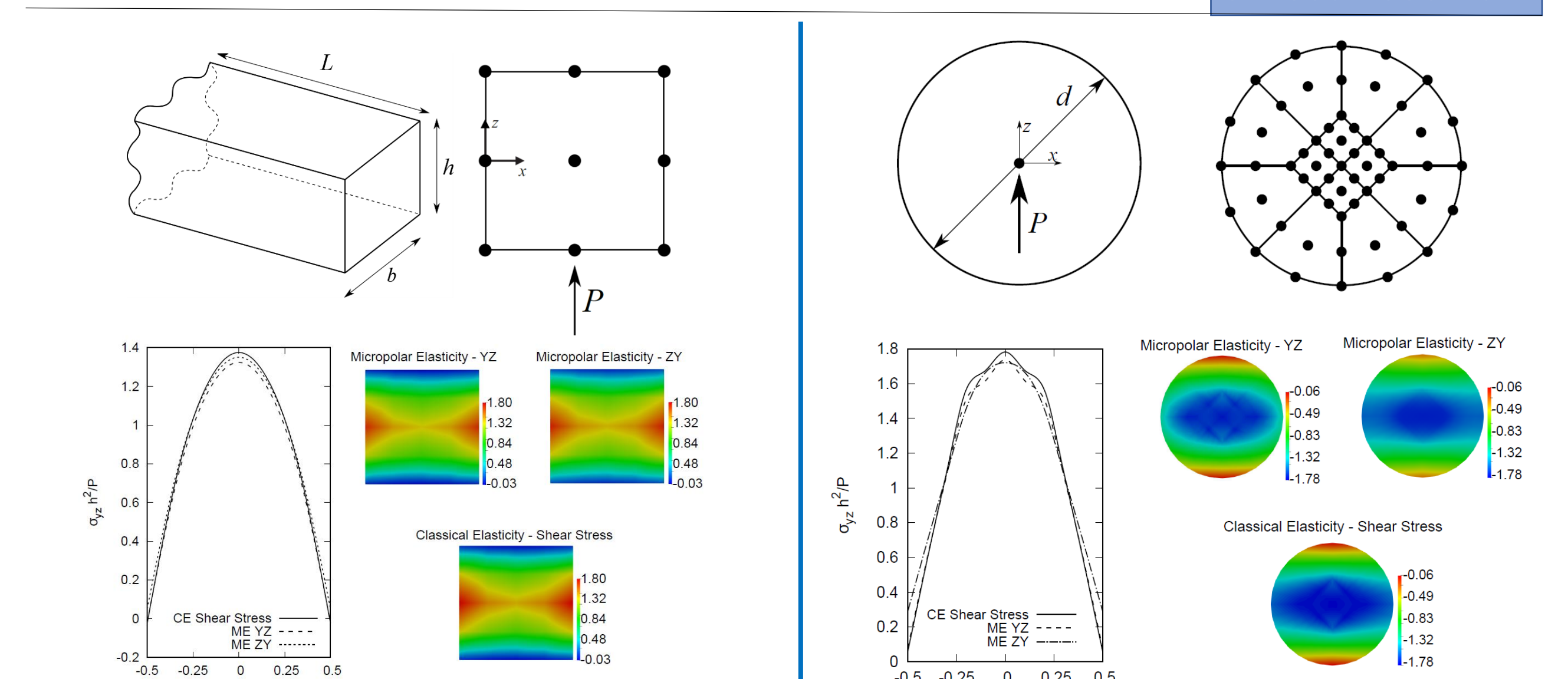
In micropolar elasticity it is assumed that the body consists of interconnected particles in the form of small rigid bodies. The internal forces are defined in terms of a classical force stress tensor  $\sigma$  and a micropolar couple stress tensor  $\mu$ :

$$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{21} & \sigma_{31} \\ \sigma_{12} & \sigma_{22} & \sigma_{32} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{bmatrix} \quad \mu = \begin{bmatrix} \mu_{11} & \mu_{21} & \mu_{31} \\ \mu_{12} & \mu_{22} & \mu_{32} \\ \mu_{13} & \mu_{23} & \mu_{33} \end{bmatrix} \quad \sigma = \lambda(tr\varepsilon)I + (\mu + \alpha)\varepsilon + (\mu - \alpha)\varepsilon^T$$

The micropolar deformation is fully described by the asymmetric strain  $\varepsilon$  and twist  $\chi$  tensors

$$\varepsilon = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{21} & \varepsilon_{31} \\ \varepsilon_{12} & \varepsilon_{22} & \varepsilon_{32} \\ \varepsilon_{13} & \varepsilon_{23} & \varepsilon_{33} \end{bmatrix} \quad \chi = \begin{bmatrix} \chi_{11} & \chi_{21} & \chi_{31} \\ \chi_{12} & \chi_{22} & \chi_{32} \\ \chi_{13} & \chi_{23} & \chi_{33} \end{bmatrix} \quad \varepsilon_{ij} = u_{i,j} + e_{ijk}\omega_k \quad i, j, k = 1, 2, 3$$

### Stress results



## Publications

- E. Carrera, A. Pagani, R. Augello, "Effect of large displacements on the free vibration of composite beams", Submitted, 2019.
- R. Augello, E. Carrera, A. Pagani, "Unified theory of structures based on micropolar elasticity", Meccanica, 1-16, 2019.
- A. Pagani, E. Carrera, R. Augello, R. Azzara, "Multibody simulation and descent control of a space lander", Submitted, 2019.
- A. Pagani, R. Augello, G. Governale, A. Viglietti, "Drop test simulations of composite leaf spring landing gears", Aerotecnica Missili & Spazio, 98(1):63-74, 2019.

## Objectives

- Extend the Micropolar Elasticity to deal with two-dimensional structure (2D), such as plates and shell, within the CUF framework.
- Apply the developed models to practical engineering problems (smart structures and biostructures);
- Adding geometrical nonlinear analysis on Micropolar models to take into account any problems in the large displacement field.
- Collaboration with City University of Hong Kong.