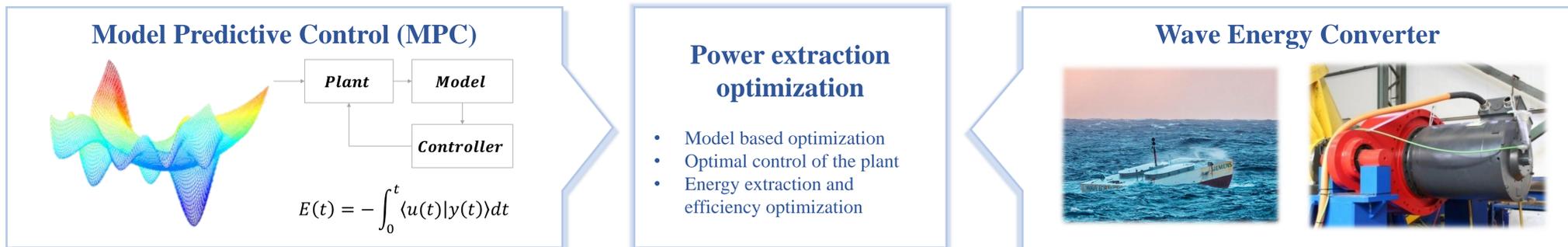


# Model Predictive Control of a Hydraulic Power Take-Off for Wave Energy Converter

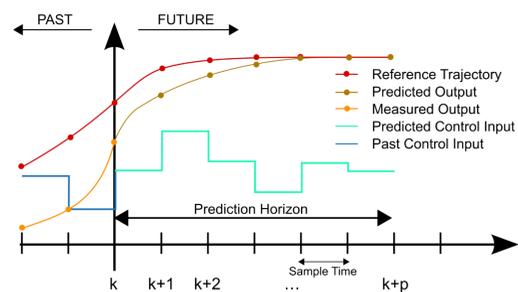
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## Project aim

Design of a Model Predictive Controller for a high efficiency PTO system for the management of renewable sources in isolated grids. The application of the MPC overcome the problem of the power extraction maximization in renewable sources performing a constrained optimization basing on the future knowledge of the plant behaviour.



## MPC philosophy



- At time k the controller solve an optimal control problem over the horizon  $[k, k + p]$
- Only the first control action  $u(k)$  is applied to the plant
- At time  $k+1$ , given new measurements, a new optimization is performed over the horizon  $[k + 1, k + p + 1]$



## Unknown input estimation

Kalman Filter Observer

- Time Update step:

$$P_{k+1}^- = AP_k A^T + Q$$

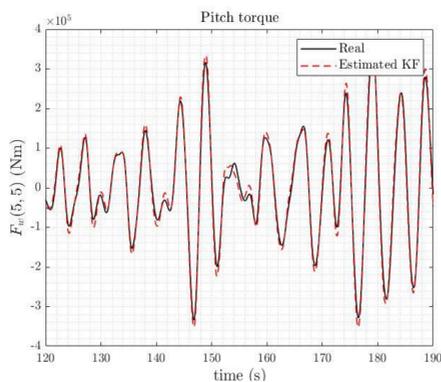
$$S_{k+1}^- = AS_k$$

- Measurement Update step:

$$K_{k+1} = P_{k+1}^- C (C P_{k+1}^- C^T + R)^{-1}$$

$$P_{k+1} = (I - K_{k+1} C) P_{k+1}^-$$

$$S_{k+1} = S_{k+1}^- + K_{k+1} (Y_{k+1} - C S_{k+1}^-)$$

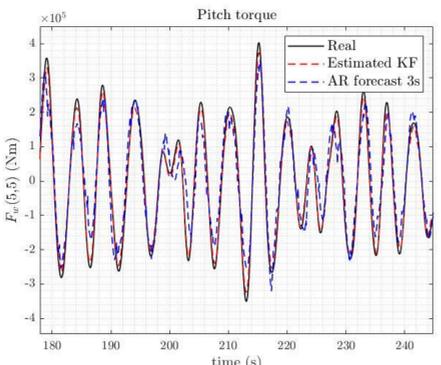


## Wave Force prediction

Autoregressive Model

$$F_t = \sum_{i=1}^p a_i \cdot F_{t-p} + \varepsilon_t \quad : t \text{ value}$$

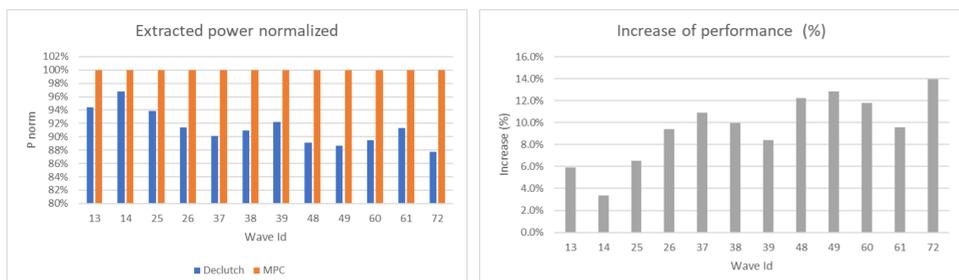
$$F_{t+n} = \sum_{i=1}^p a_i \cdot F_{t-p+n} + \varepsilon_t \quad : t + n \text{ values}$$



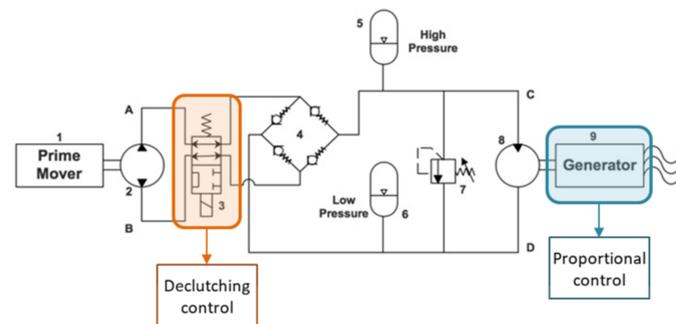
## MPC optimization

Non convex – constrained optimization

Comparison between a simple declutching control and MPC



## PTO architecture and control framework



- Dynamic equations:

$$\dot{X}(t) = f(X(t)) + b(X(t))u(t) + Gv(t)$$

$$y(t) = CX(t)$$

The system is linear in respect of the control  $u$

- Cost function:

$$J(X(t), u(t), v(t), t) = - \int_0^{t_f} \langle u(t), y(t) \rangle dt$$

- Pontryagin's maximum principle:

$$H(X, u, v, \lambda, t) = (y(t) + \lambda^T b(X(t))) u(t) + \lambda^T (f(X(t)) + Gv(t))$$

We obtain  $\rightarrow H(X, u, v, \lambda, t) = \Phi u + \lambda^T \Theta$

The Hamiltonian is linear in respect of the control  $u$

Optimal control problem

$$H(X, u, v, \lambda, t) = \Phi u + \lambda^T \Theta$$

$$X(0) = X_0$$

$$\lambda(t_f) = 0$$

$$\frac{\partial \lambda}{\partial X} = \dot{\lambda}$$

$$\frac{\partial H}{\partial u} = \Phi$$

$$u = \begin{cases} u_{max} & \text{if } \Phi < 0 \\ u_{sing} & \text{if } \Phi = 0 \\ u_{min} & \text{if } \Phi > 0 \end{cases}$$

## Future works

