Mathematical modelling, control and dynamic analysis of point absorber wave energy converter technology

Admission to the final exam – 33rd cycle

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Turin , 28/11/2020

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State of the art

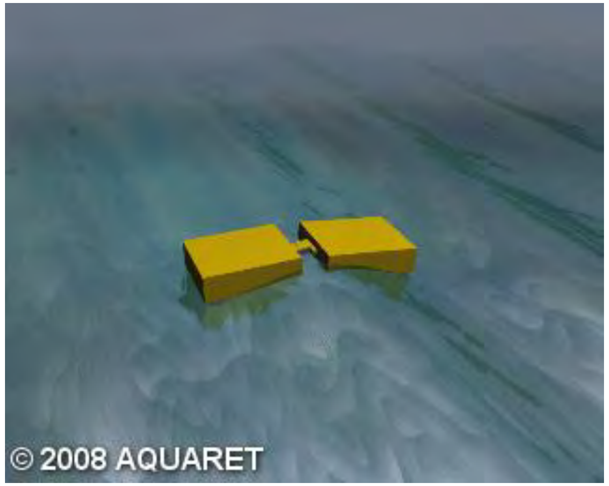
WEC Classification

Motivation

State of the art

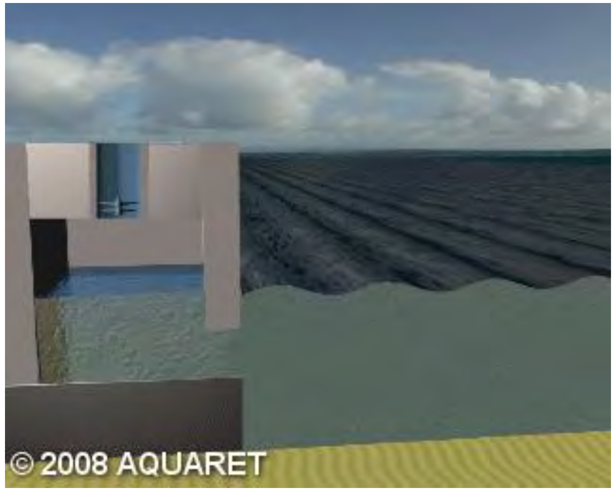
WEC Classification

Attenuator

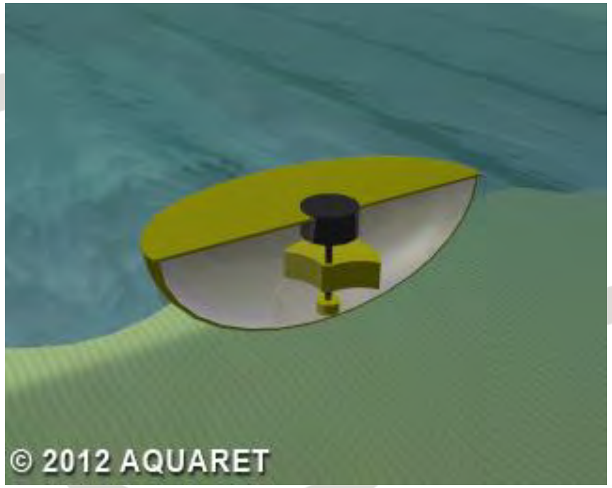


Overtopping

Oscillating Water Column



Rotating mass



Point absorber

Source: EMEC – website: http://www.emec.org.uk/

Submerged point absorber

- Wave energy absorption from all directions
- Oscilation in all degrees of freedom
- Simple mooring system design
- Elevated survival capacity
- Zero visual impact

CETO is a fully submerged, point absorber developed by Carnegie

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Mathematical model based on the potential flow theory

Cummins equation

Simulink model

Comparison with Ansys Aqwa

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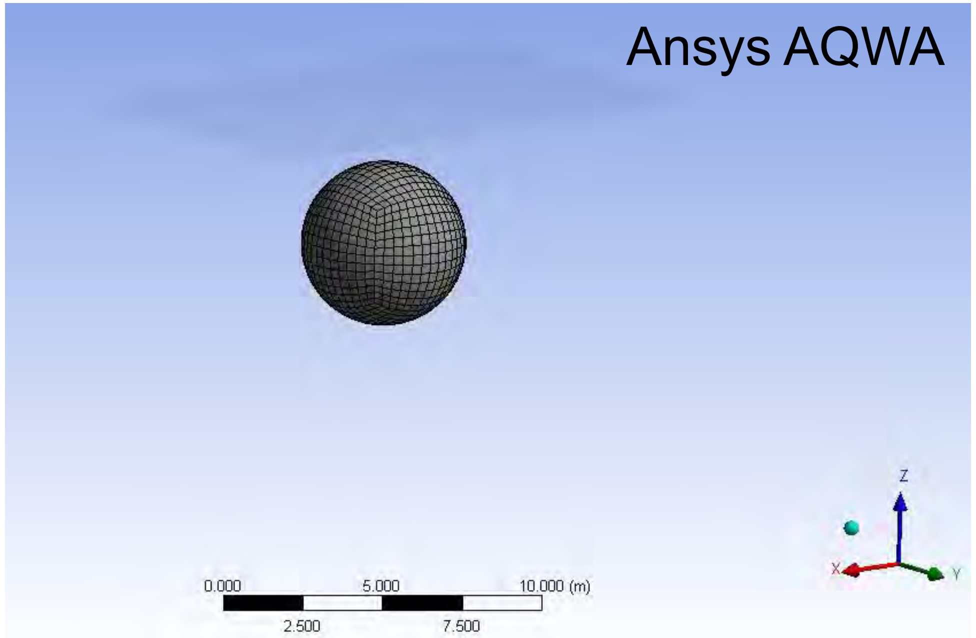
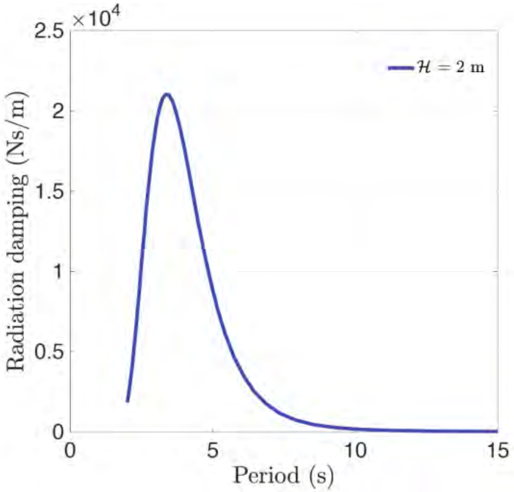
Potential flow theory

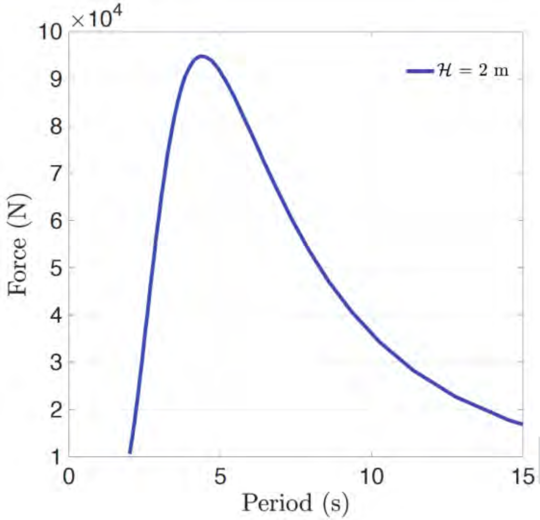
Potential flow based numerical model

Cummins equation: $(M + A_\infty)\ddot{x}(t) + \int_0^t K_r(t-\tau)\dot{X}(t)d\tau + F_m = F_e + F_{drag} + F_{pto} + F_H$

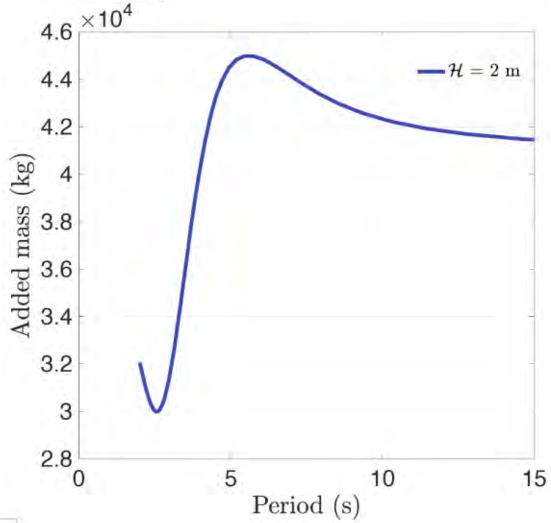




Radiation damping



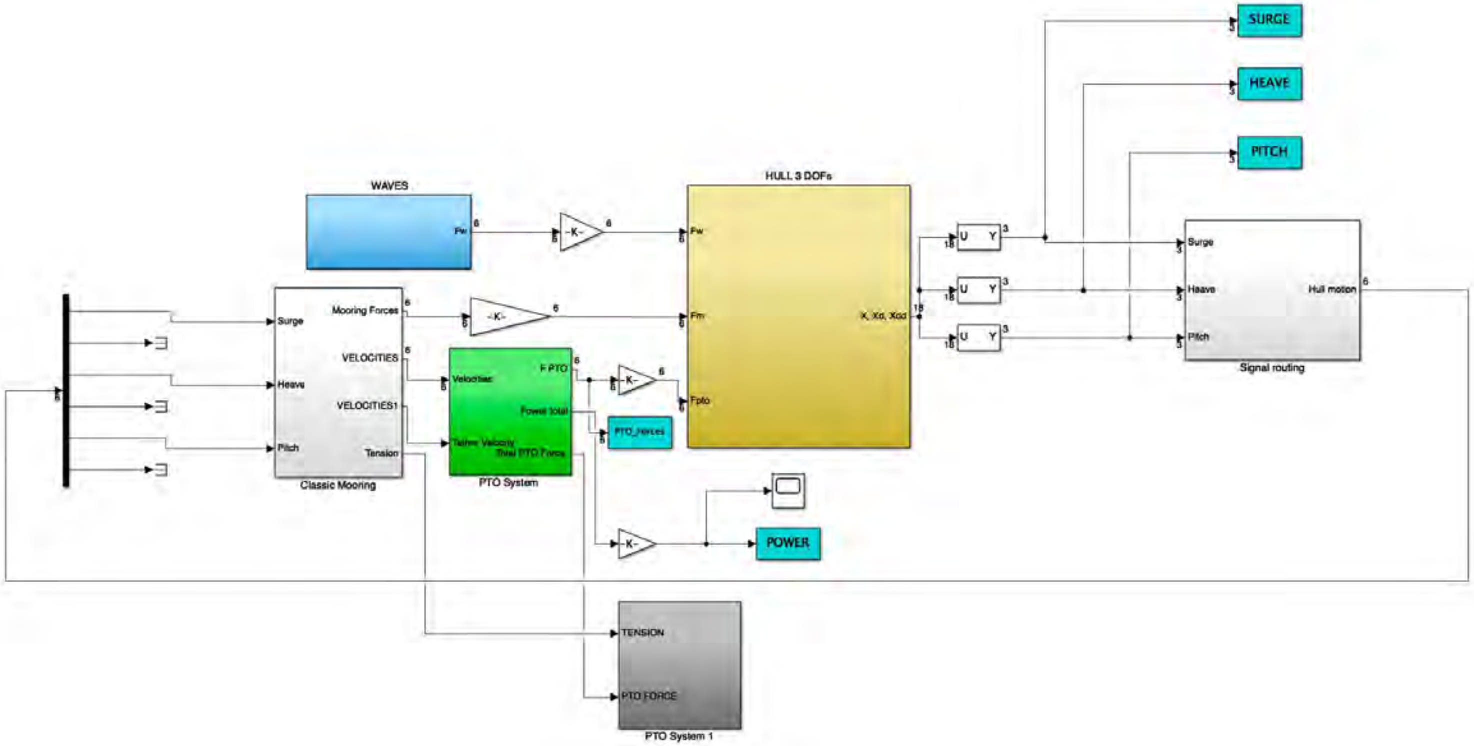
Excitation Forces



Added mass

Potential flow theory

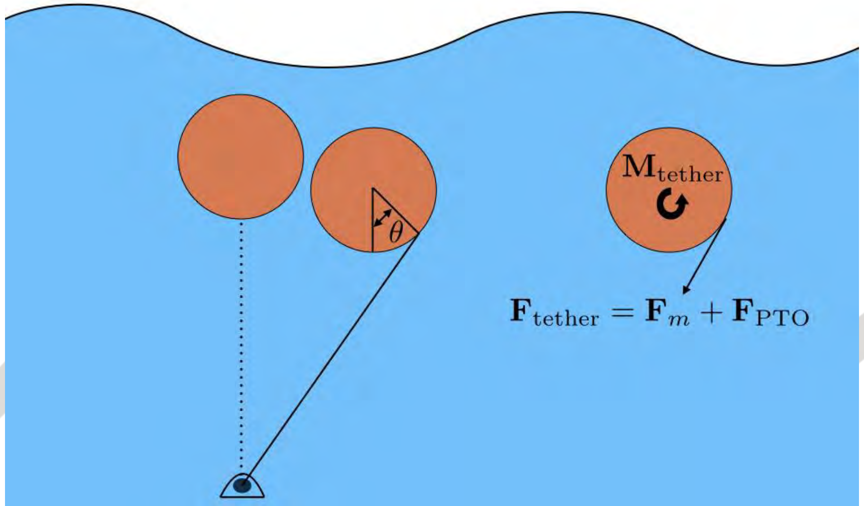
Simulink model



PTO Forces

$$F_m = k\ \Delta L$$

$$F_{PTO} = b_{pto}\frac{d(\Delta L)}{dt}$$



Linear reactive control

$$k = \ \omega^2(m + A(\omega))$$

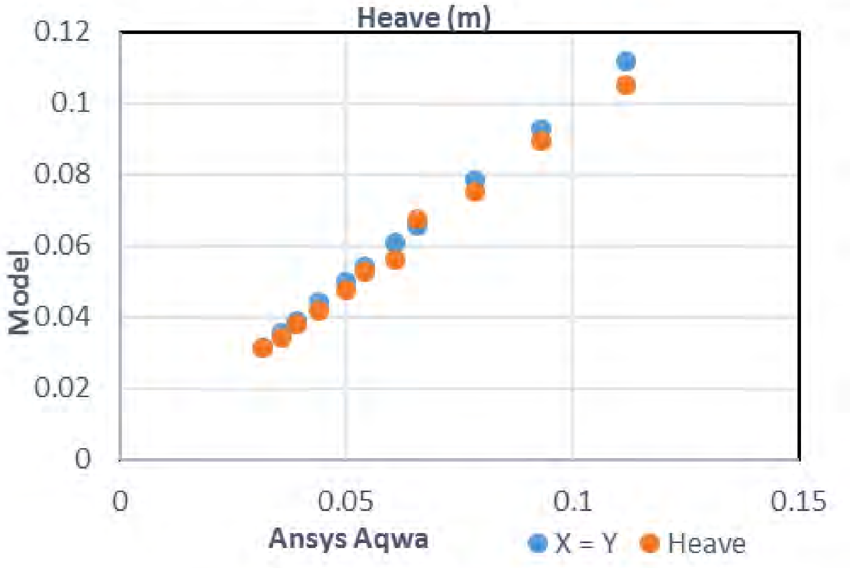
$$bpto = B(\omega)$$

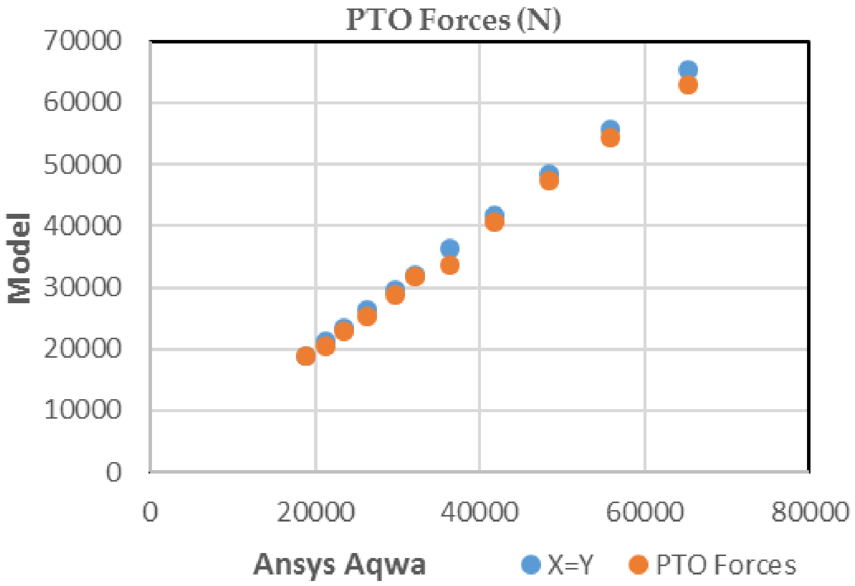
Potential flow theory

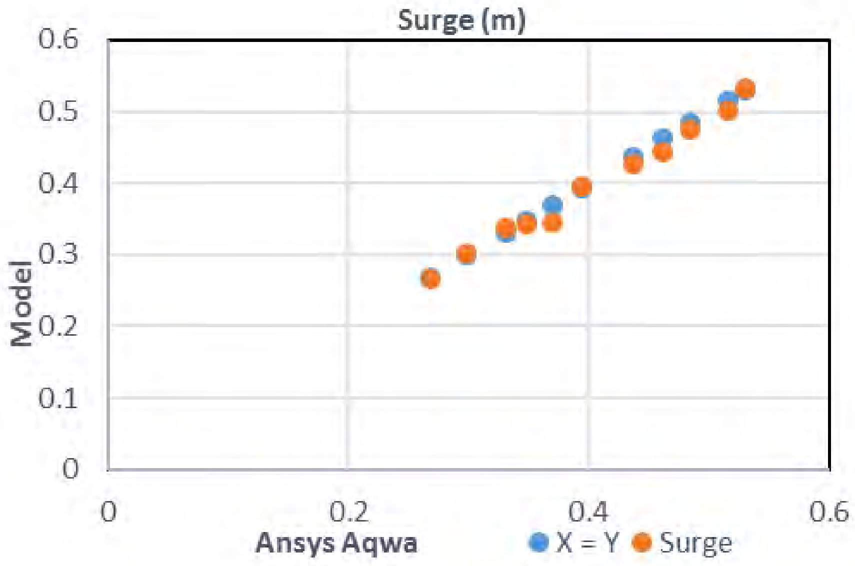
Comparison

Time domain simulations

- o Regular waves
- o Wave height: 1 m
- o Linear mooring stiffness
- o Wave periods: [5 – 10] s
- o Diameter: 5.16 m







Computational fluid dynamics analysis

Fully resolved model

Model comparison

Results and discussion

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CFD analysis

Fully-resolved model

Fully Eulerian Brinkman penalization method → IBAMR library

Momentum equation

$$\frac{\partial \rho \mathbf{u}(\mathbf{x},t)}{\partial t} + \nabla \cdot \rho \mathbf{u}(\mathbf{x},t)\mathbf{u}(\mathbf{x},t) = -\nabla p(\mathbf{x},t) + \nabla \cdot \left[\mu\left(\nabla \mathbf{u}(\mathbf{x},t) + \nabla \mathbf{u}(\mathbf{x},t)^T\right)\right] + \rho \mathbf{g} + \mathbf{f}_c(\mathbf{x},t)$$

Continuity equation

$$\nabla \cdot \mathbf{u}(\mathbf{x},t) = 0$$

Brinkman penalized constraint force

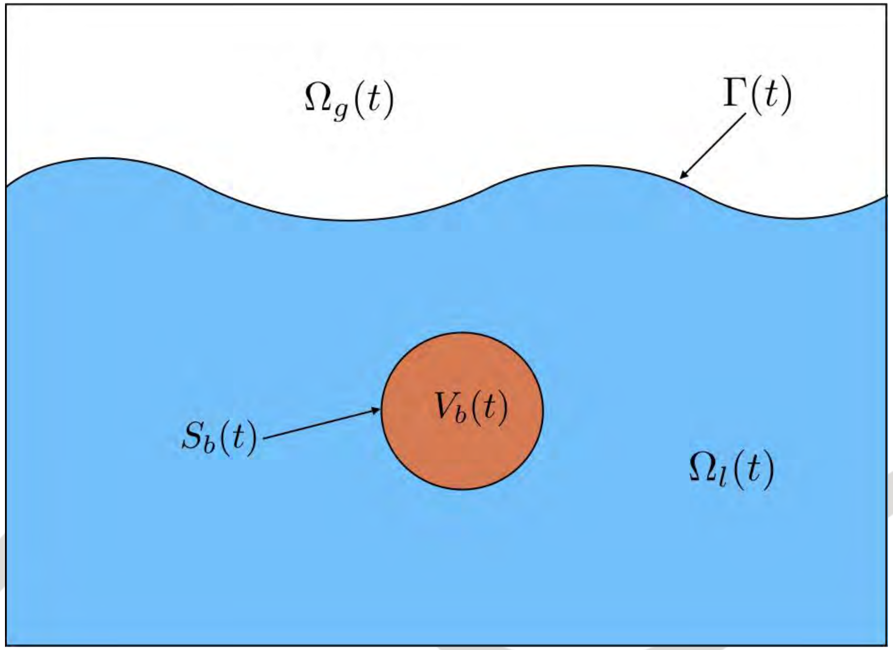
$$\mathbf{f}_c(\mathbf{x},t) = \frac{\chi(\mathbf{x},t)}{K}\left(\mathbf{u}_b(\mathbf{x},t) - \mathbf{u}(\mathbf{x},t)\right).$$

CFD analysis

Fully-resolved model

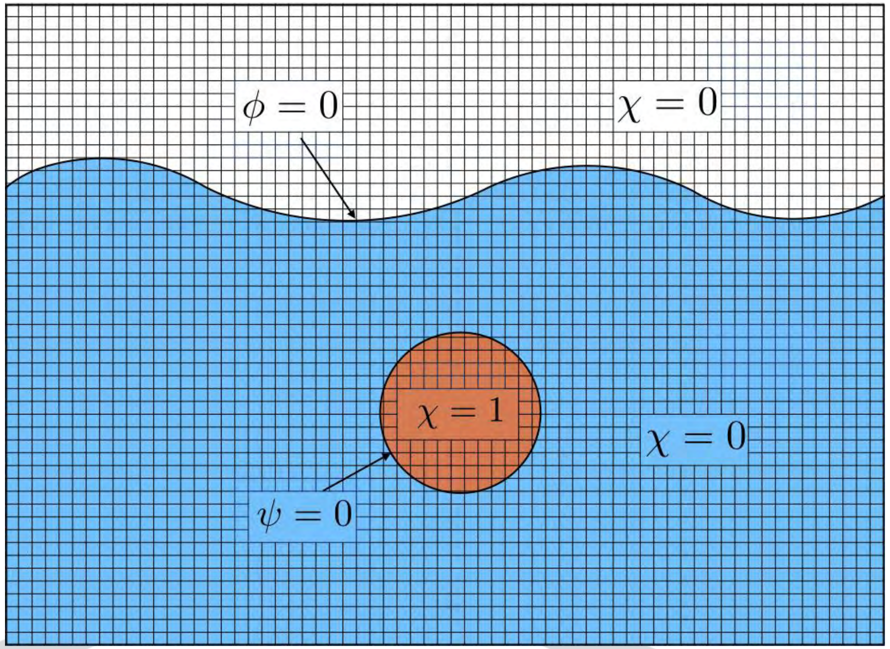
Material properties

$$\rho(\mathbf{x},t) = \rho(\phi(\mathbf{x},t),\psi(\mathbf{x},t))$$
$$\mu(\mathbf{x},t) = \mu(\phi(\mathbf{x},t),\psi(\mathbf{x},t))$$



Advection of level set fields

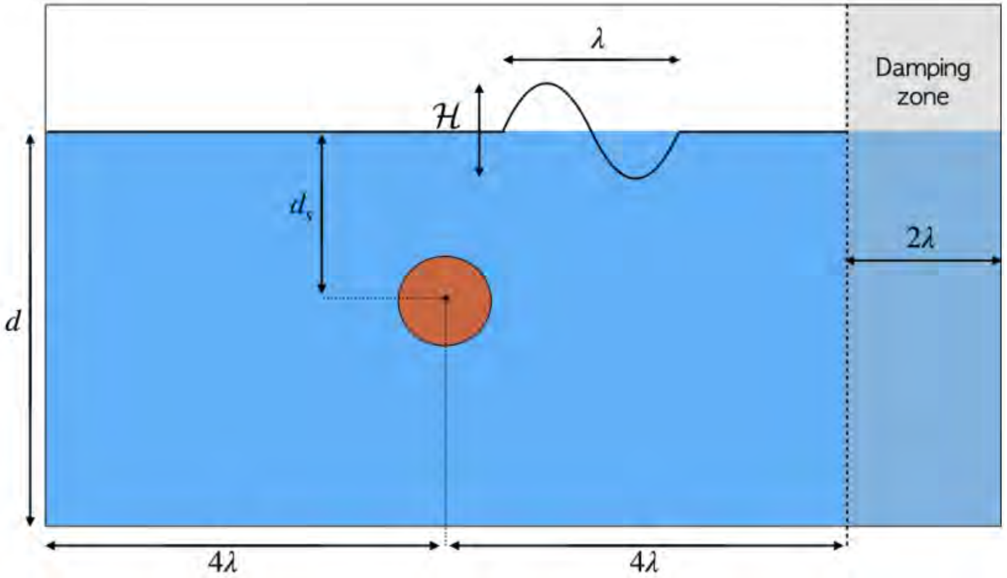
$$\frac{\partial \phi}{\partial t} + \nabla \cdot \phi \mathbf{u} = 0$$
$$\frac{\partial \psi}{\partial t} + \nabla \cdot \psi \mathbf{u} = 0$$

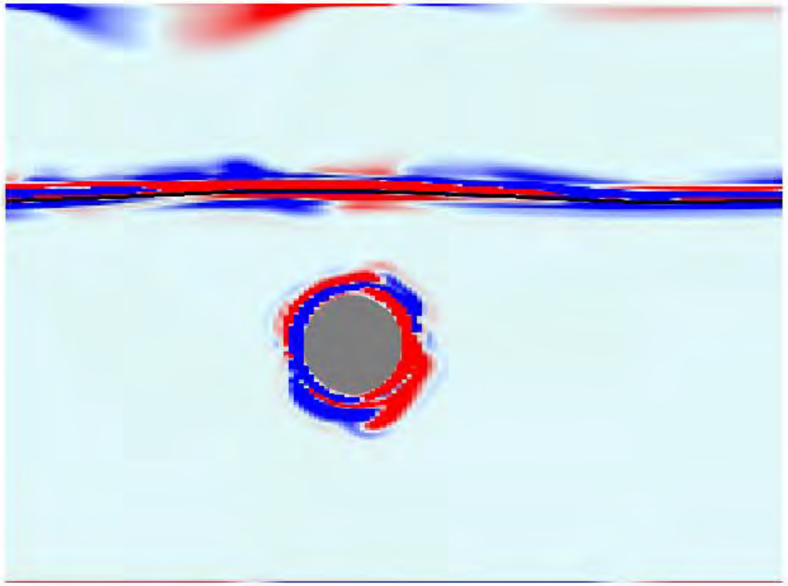


CFD analysis

Fully-resolved model

Numerical wave tank





Characteristics

$$\lambda = 1.216\ m$$

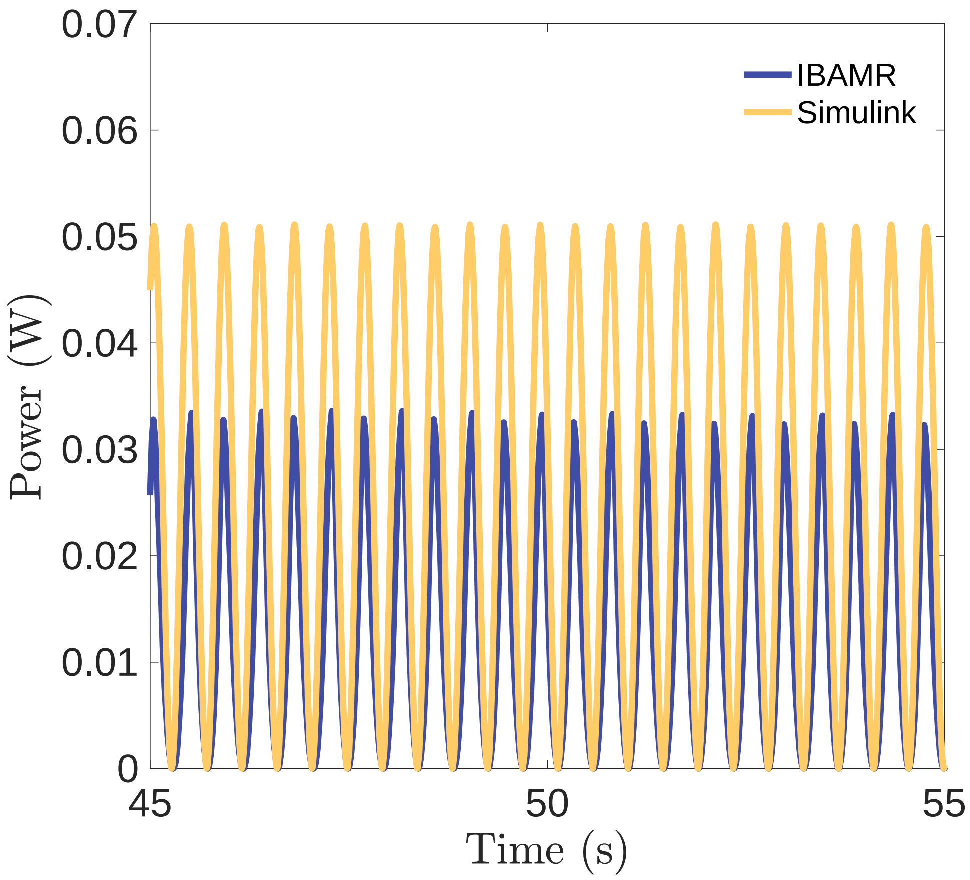
$$H = 0.01\ m$$

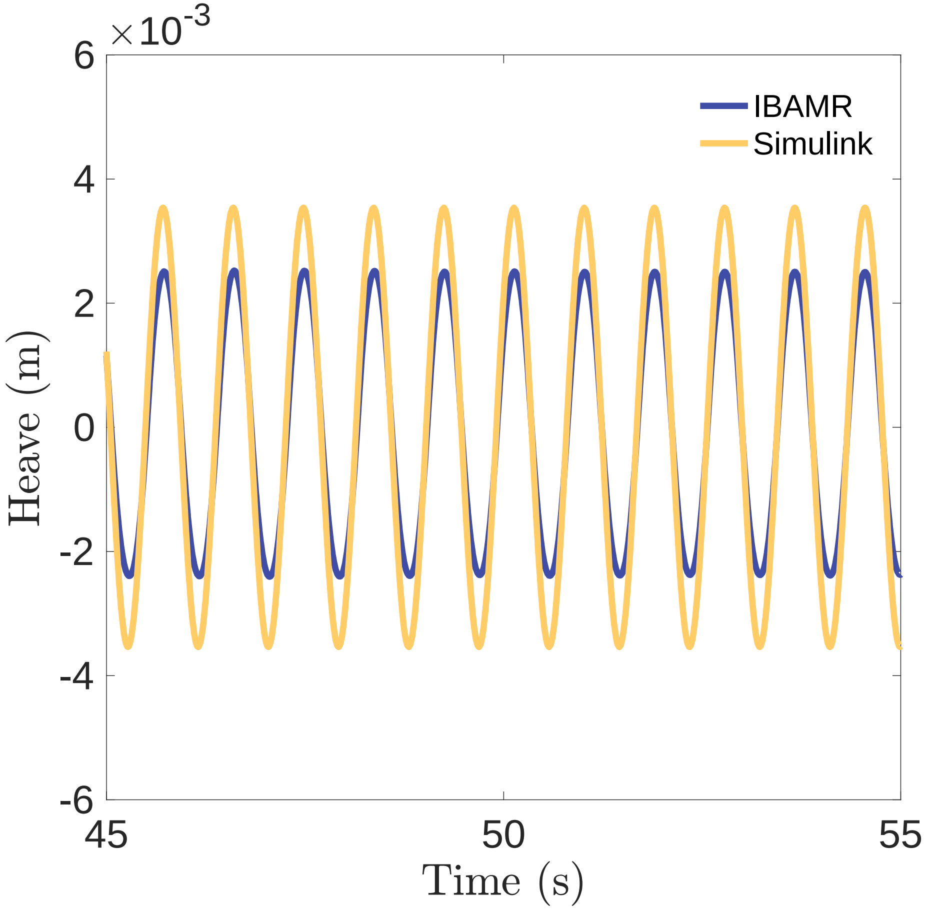
$$T = 0.8838\ s$$

$$d = 0.65\ m$$

$$d_s = 0.25\ m$$

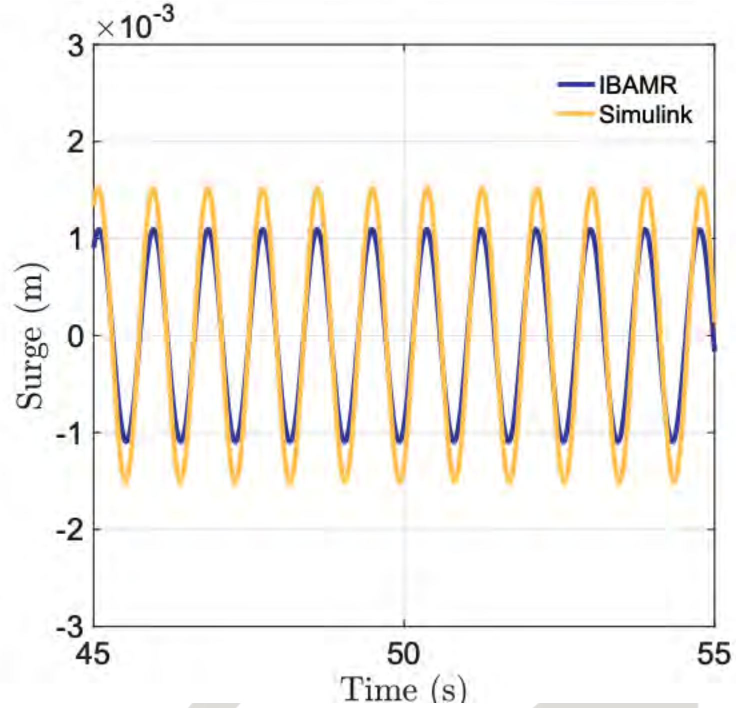
Comparison 1 DOF

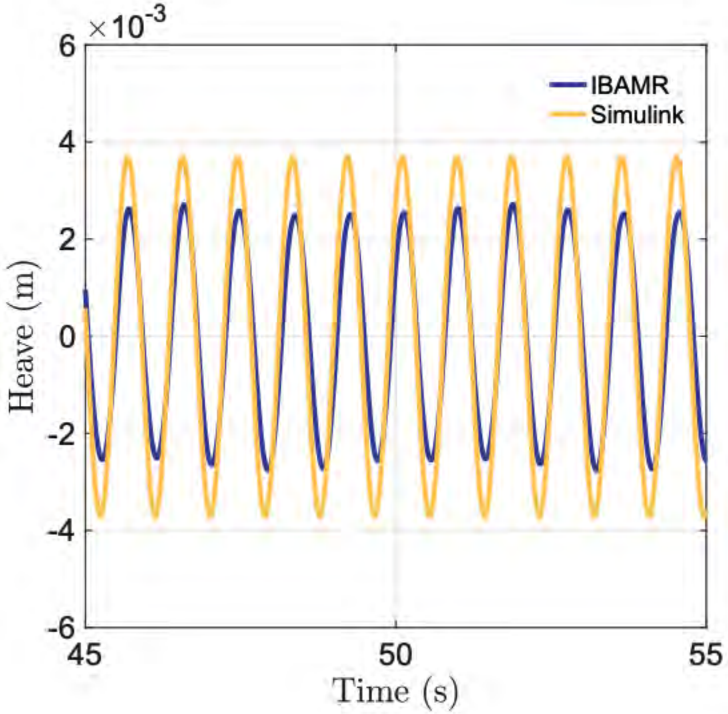


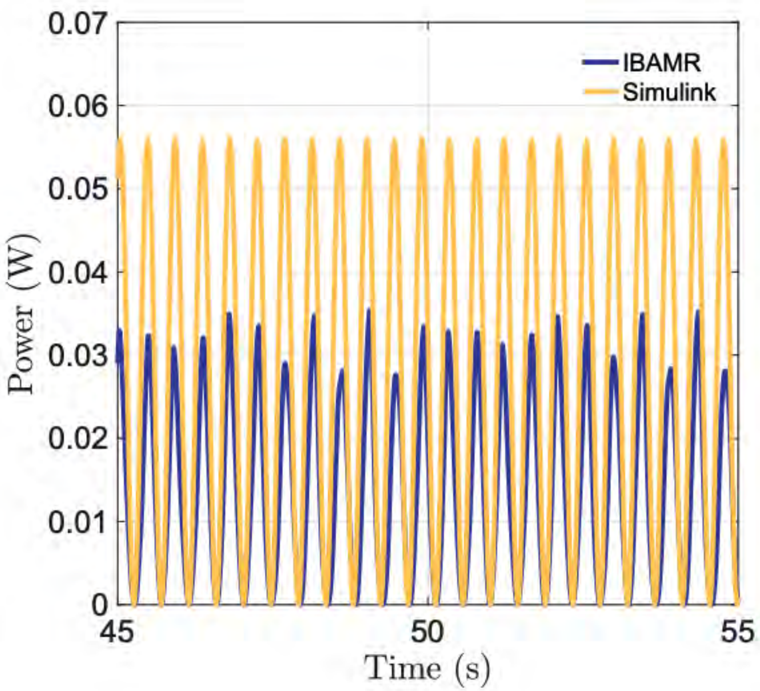


CFD analysis

Comparison 2 DOF







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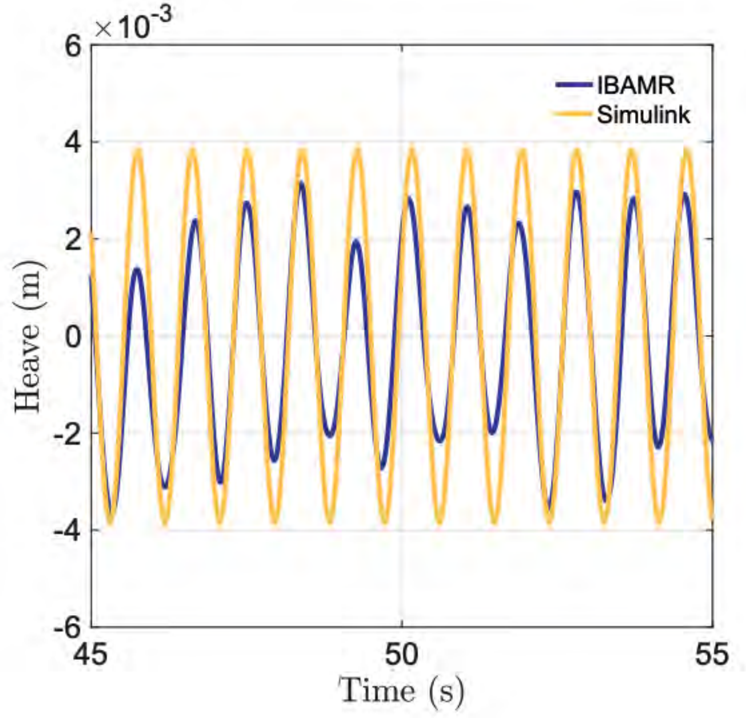
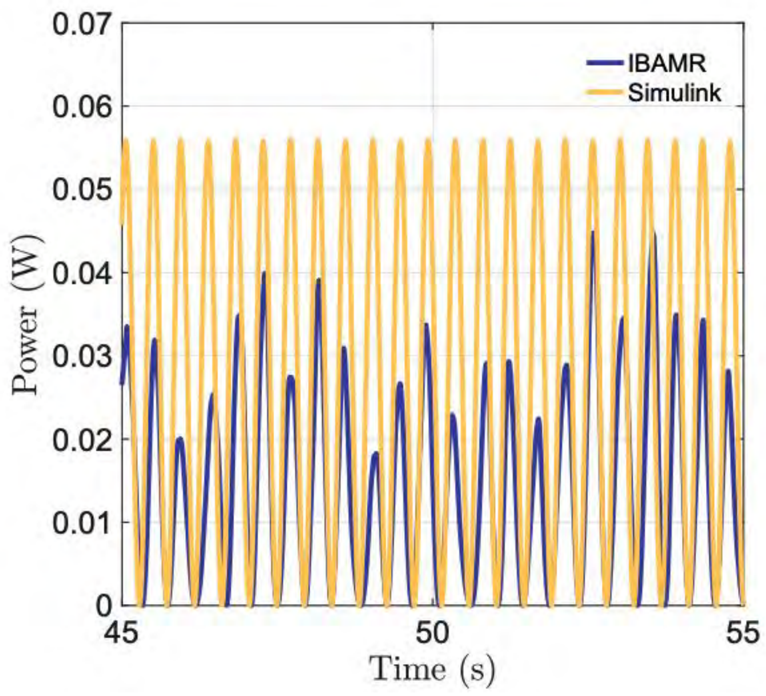
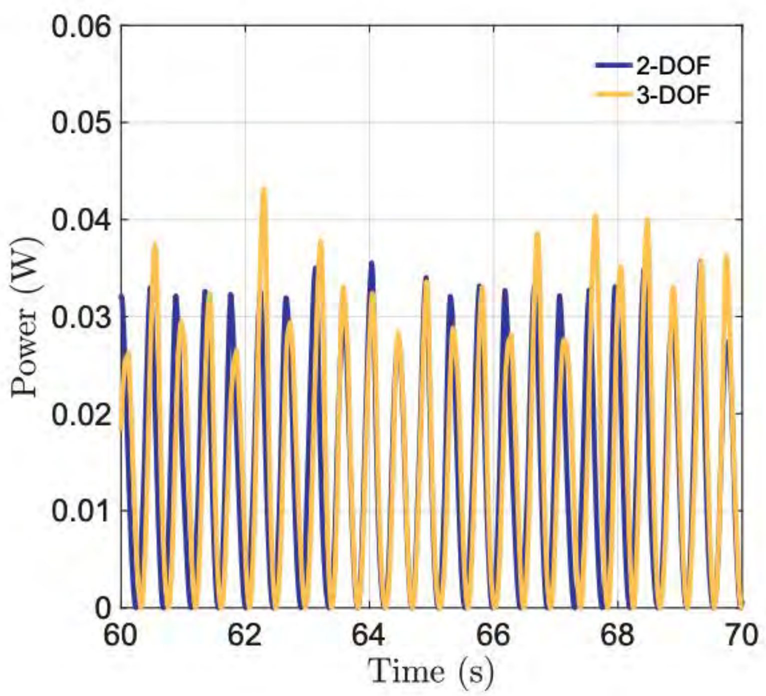
CFD analysis

Model Comparison

Comparison 3 DOF







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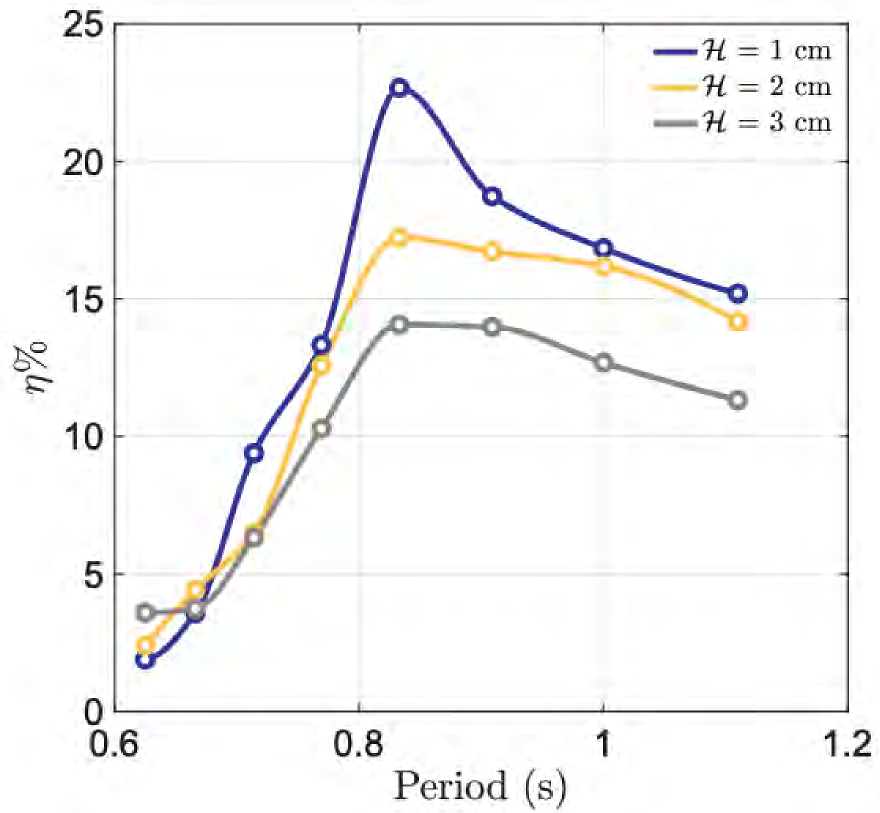
CFD analysis

Results

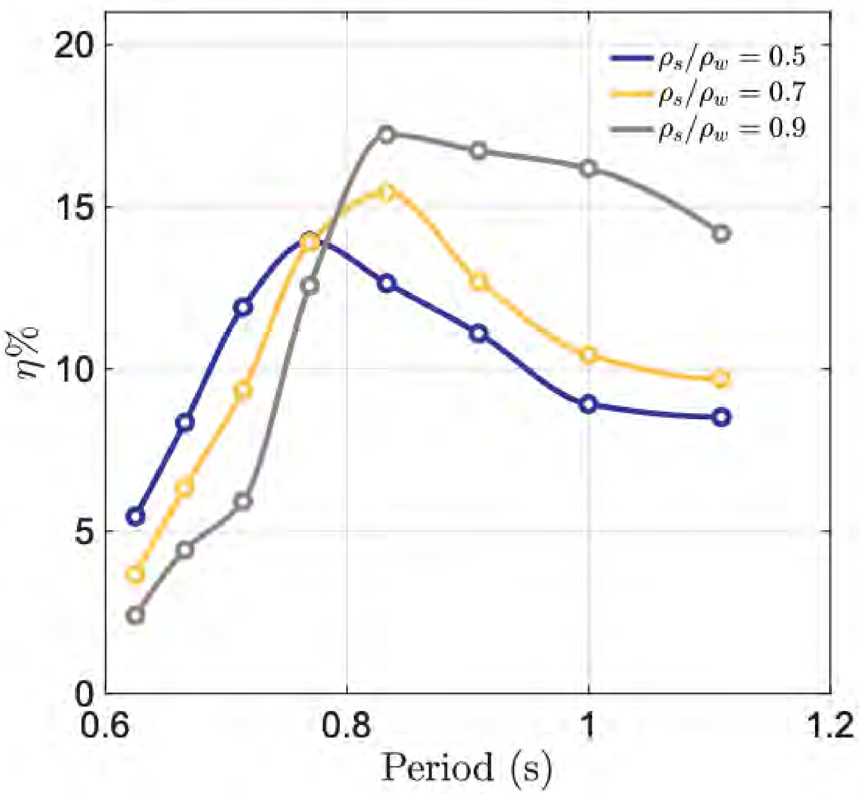
Efficiency

$$\eta = \frac{\overline{P}_{\text{absorbed}}}{\overline{P}_{\text{wave}}} = \frac{\frac{1}{\mathcal{T}}\left[\int_t^{t+\mathcal{T}} P_{\text{absorbed}}(t)\,\mathrm{d}t\right]}{\frac{1}{8}\rho_w g \mathcal{H}^2 c_g}$$

Wave height influence



Buoy density



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Experimental Campaign

Experimental setup

Small scale prototype

Results

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Experimental campaign

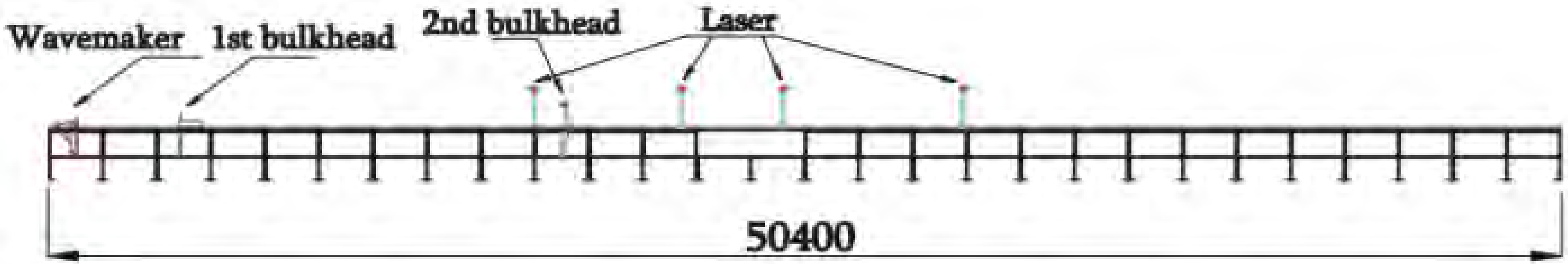
Setup

<u>Hydraulics laboratory</u>

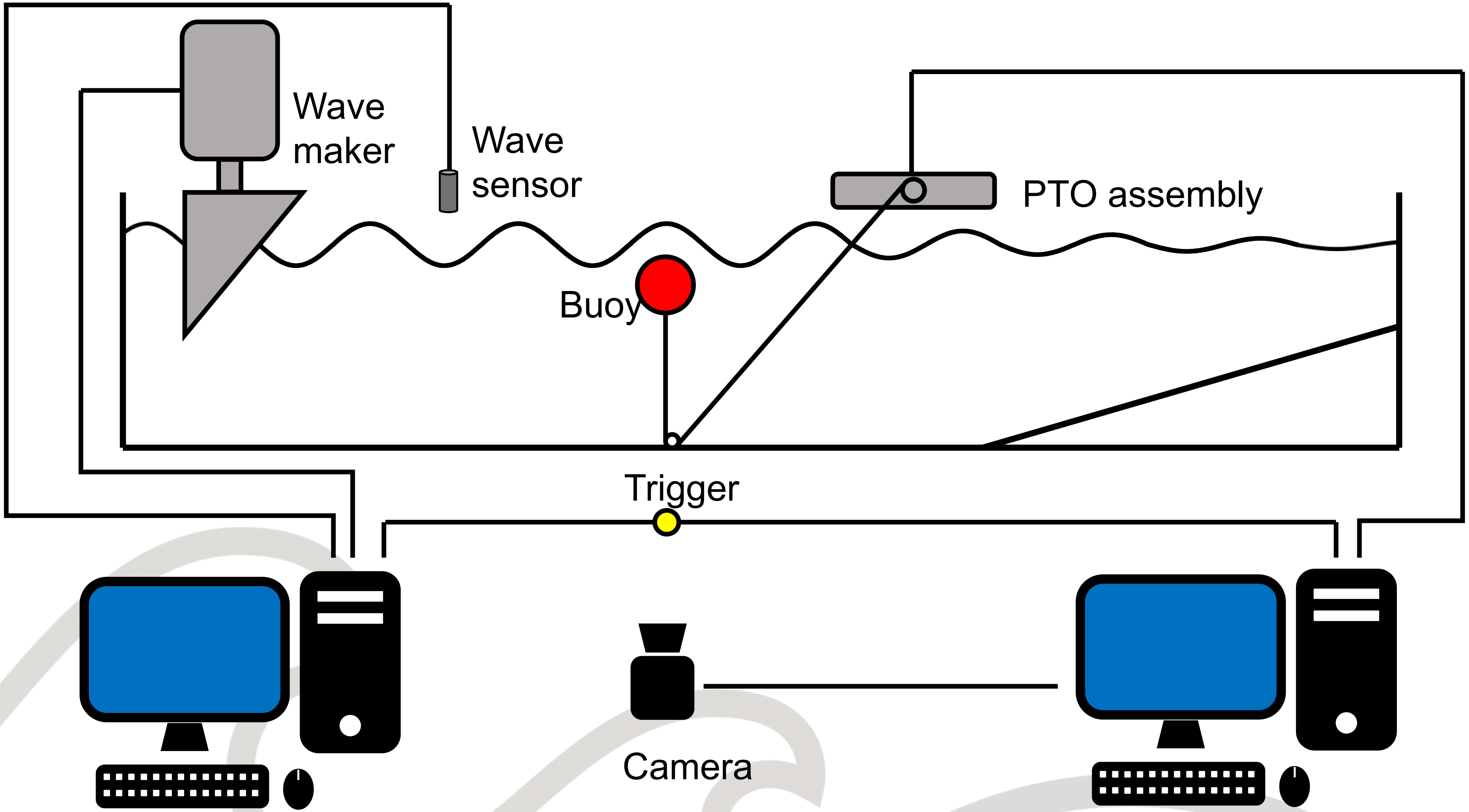
Wave flume

Wave maker

Wave sensors

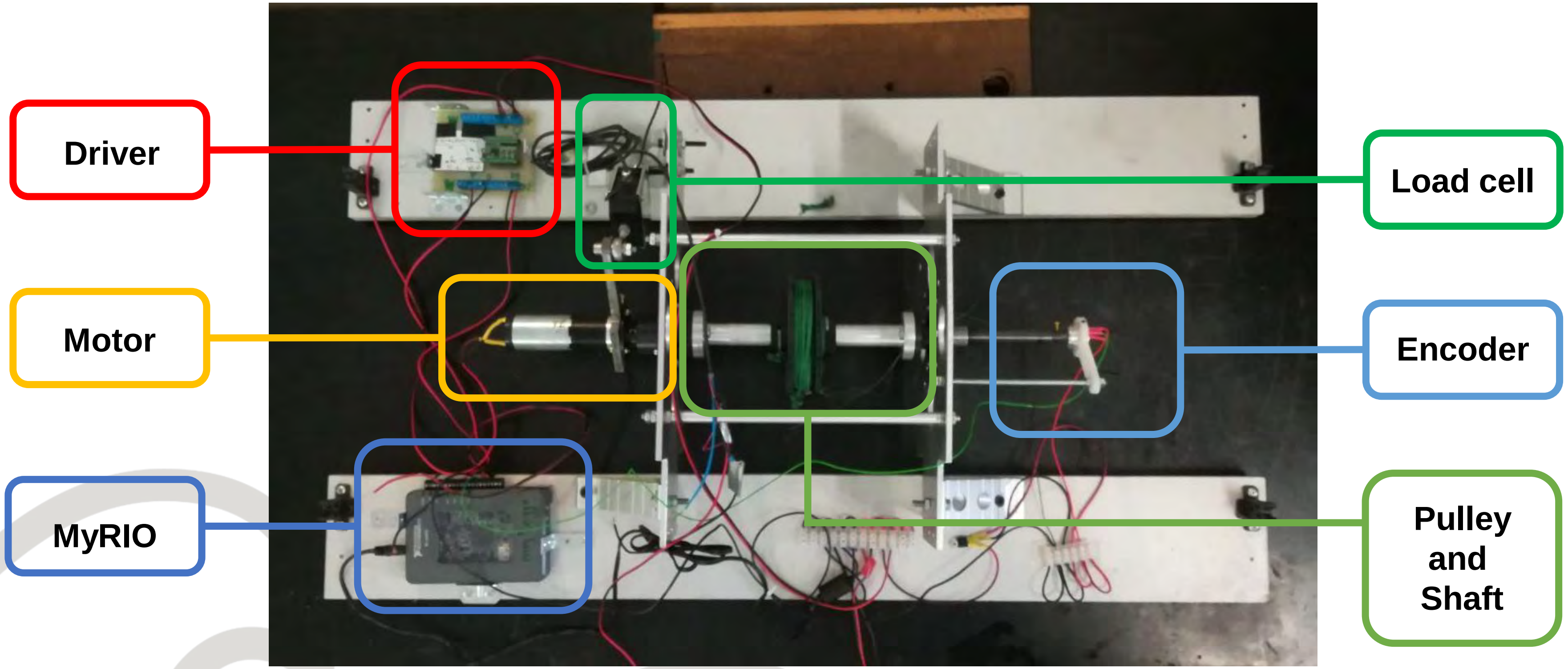


Experimental campaign



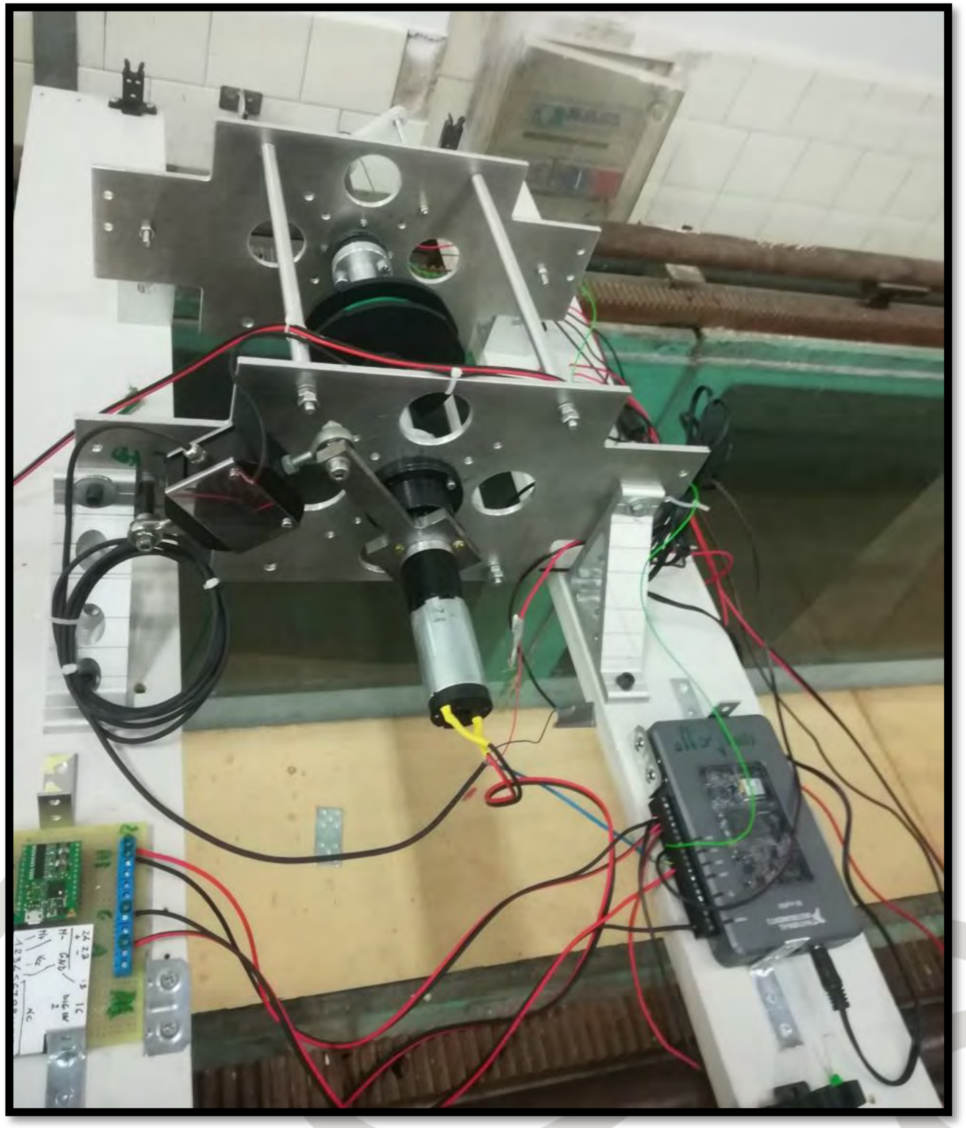
Experimental campaign

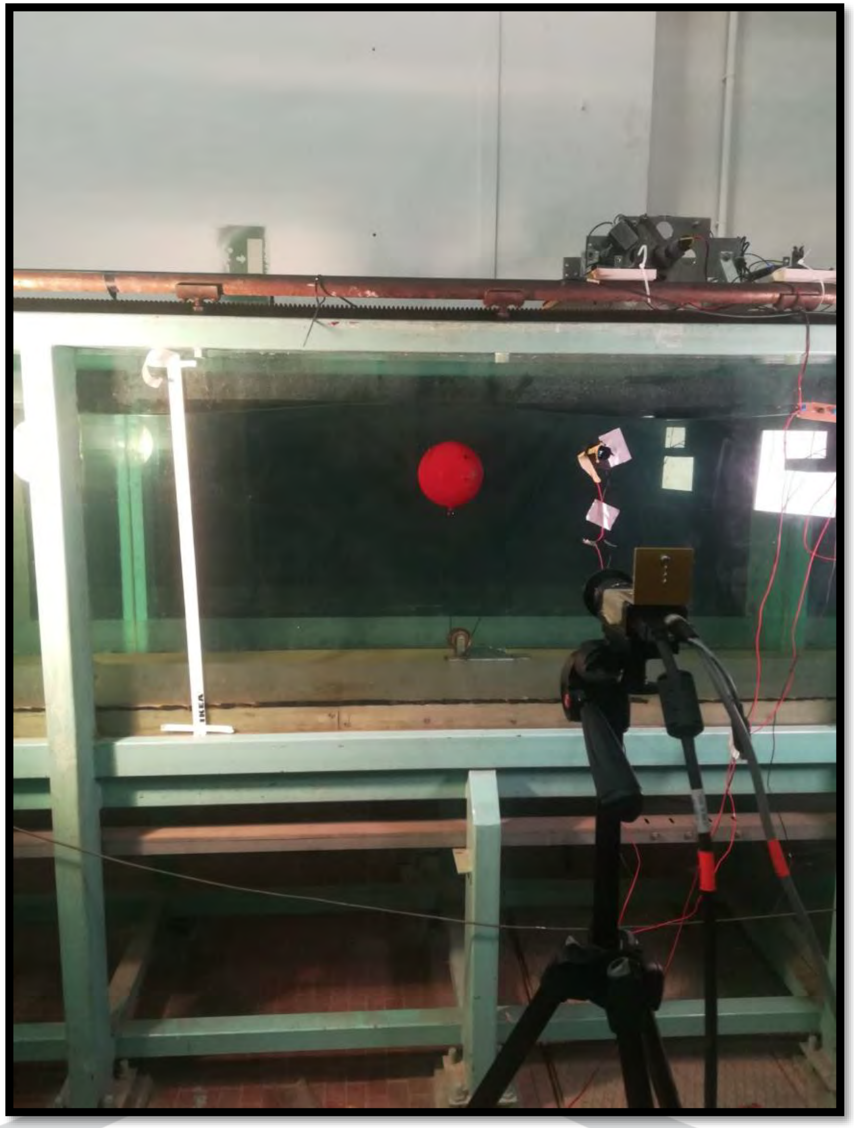
Prototype



Experimental campaign

Prototype





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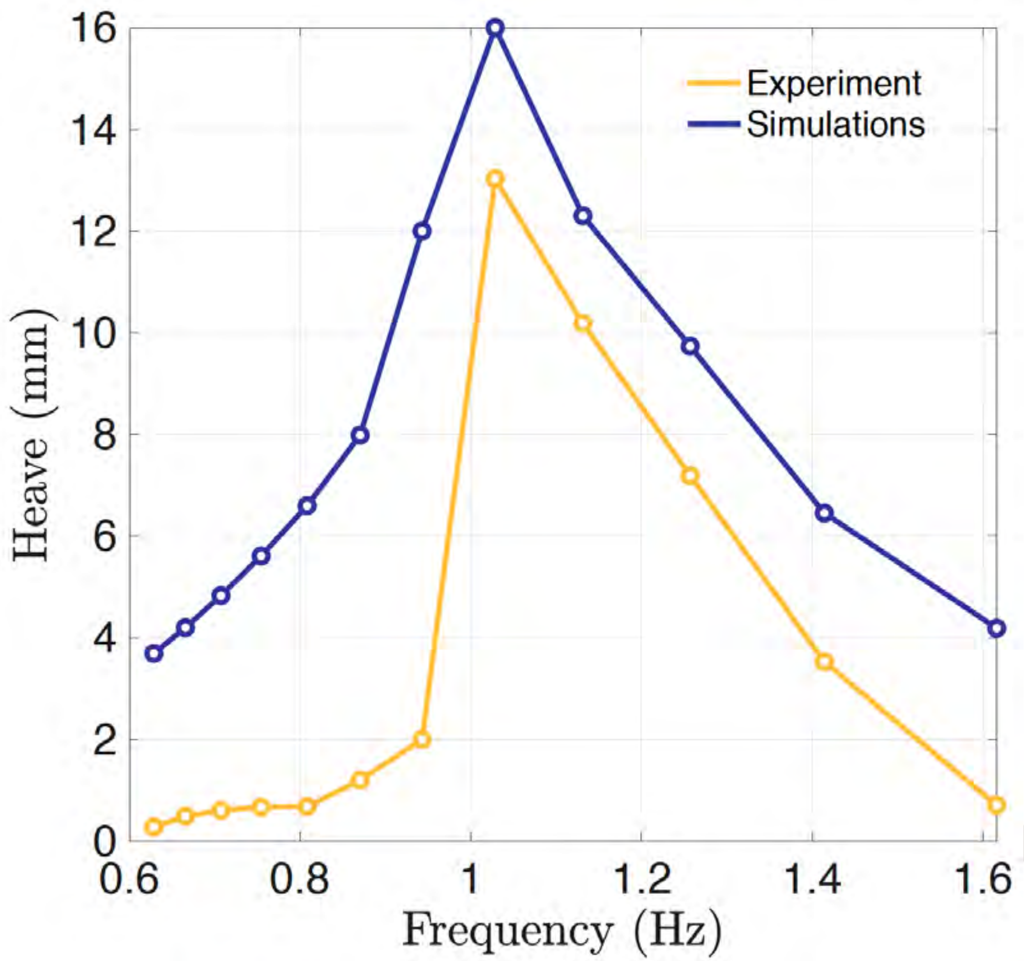
Experimental campaign

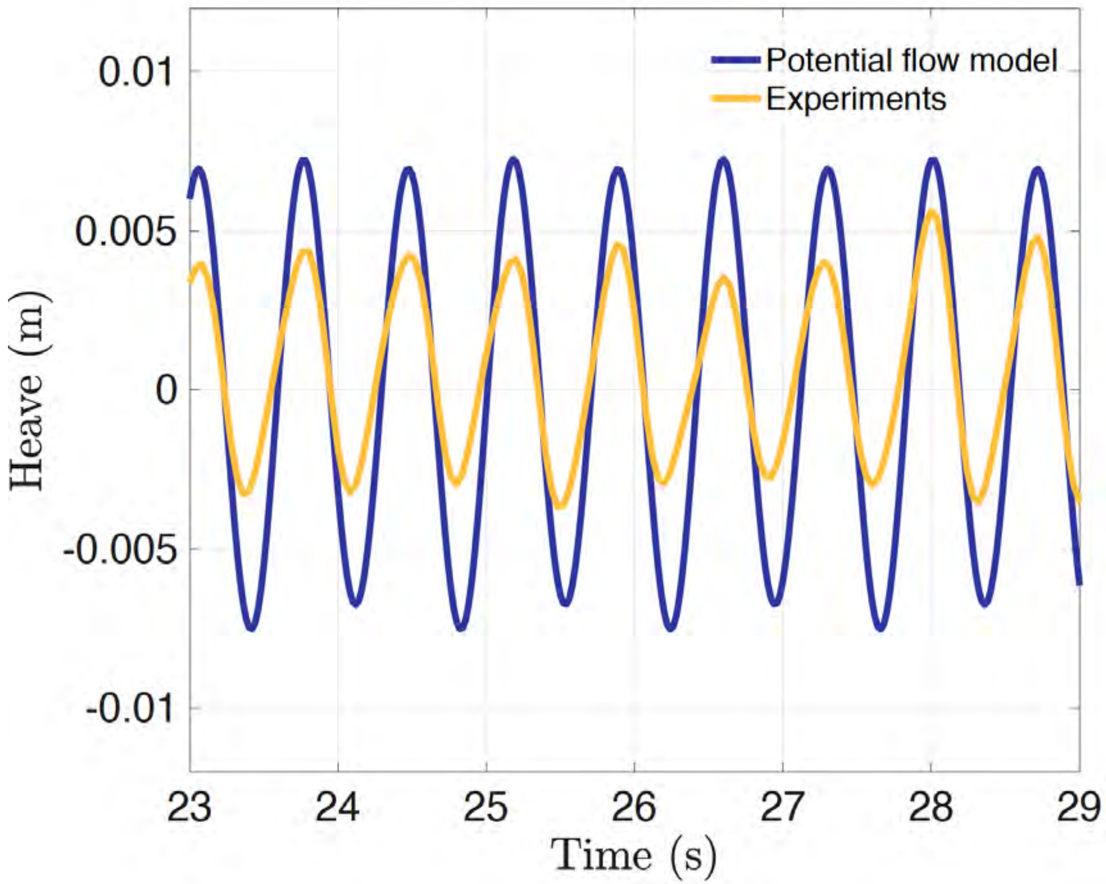
Froude scaling

Hull 2	λ	32
	λ^2	1024
	λ^3	32768
	Root λ	5.656854249
	λ^(7/2)	185363.8
Characteristics	**Real Case - Pantelleria**	**Scale Experiment (1:32)**
Wave Height (m)	1	0.0312
Smallest Period (s)	3	0.53
Highest Period (s)	10	1.767
Highest Frequency (Hz)	0.33	1.885
SmallestFrequency (Hz)	0.1	0.565
Radius (m)	2.58	0.080
Diameter (m)	5.14	0.161
Volume (m^3)	71.64	0.0021
Mass (kg)	66088	2.0168
Density (kg/m^3)	922.5	922.5

Experimental campaign

Time domain comparison





Frequency = 1.41 Hz

Design process

Pantelleria

Methodology

Budal diagram

Results

Design process

Pantelleria

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Design process

Pantelleria

Island of Pantelleria

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Design process

Methodology

Methodology proposed by Falnes

- Wave power threshold $J_T(kW/m)$ which is being exceeded only one third of the year
- Peak period of the the most frequent waves
- Determine the wave height $J_T = \frac{\rho g^2 H^2 T}{32\pi}$

Design process

Methodology proposed by Falnes

- Wave power threshold $J_T(kW/m)$ which is being exceeded only one third of the year
- Peak period of the the most frequent waves
- Determine the wave height $J_T = \frac{\rho g^2 H^2 T}{32\pi}$

$$\boxed{J_T = 5{,}048\ kW/m}$$

$$\boxed{H = 1\ m}$$

$$\boxed{T = 5.57\ s}$$

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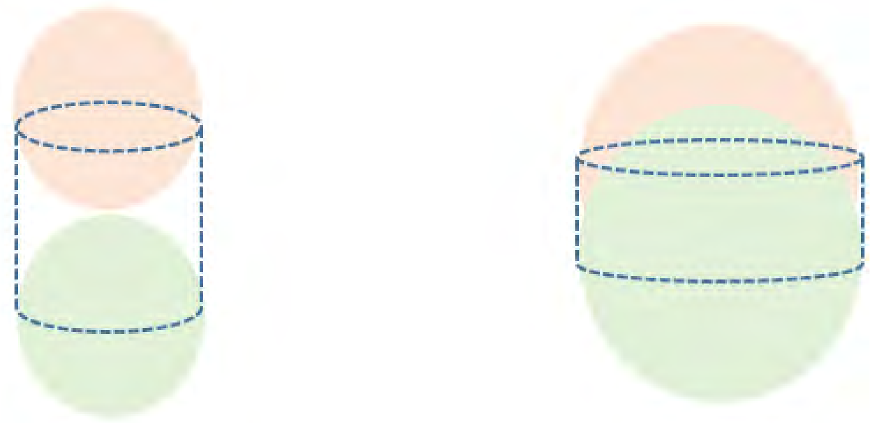
Design process

High frequency limit

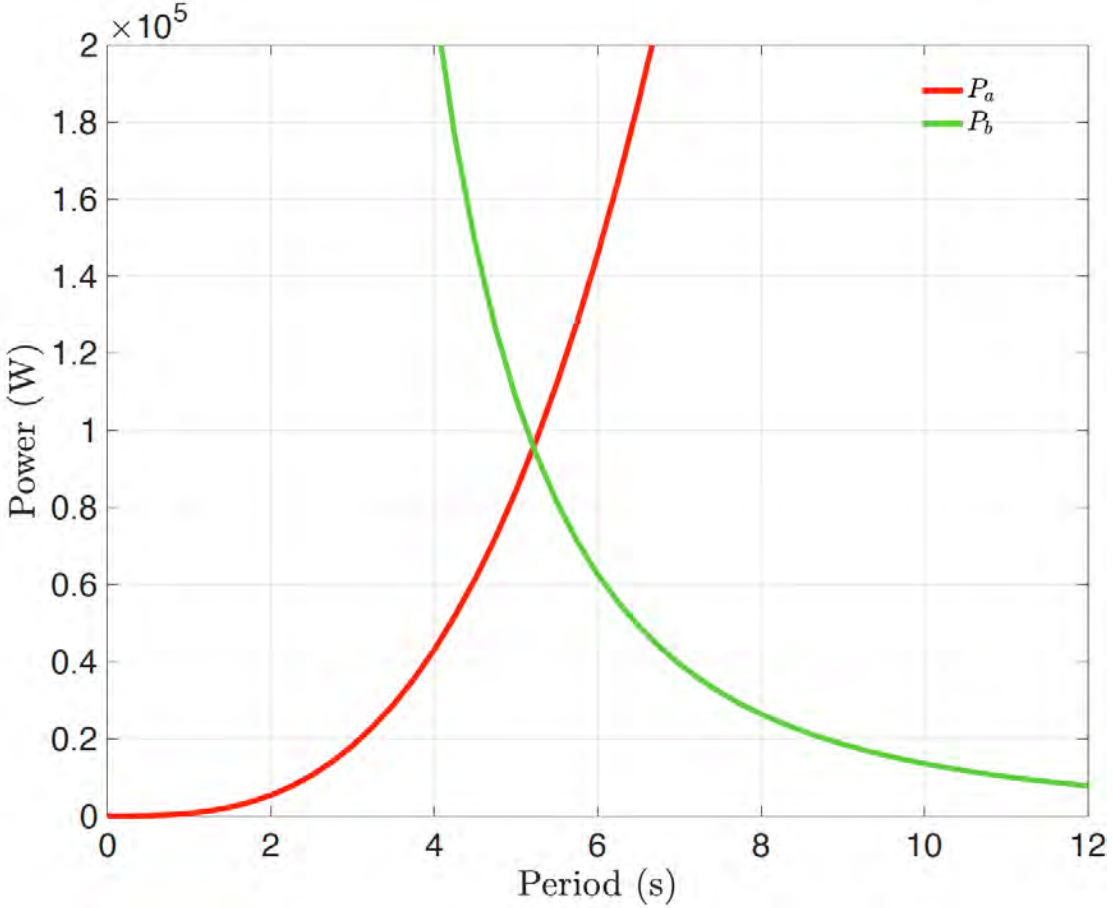
$$P_a = c_\infty T^3 H^2$$

$$V = bV_s$$



Low frequency limit

$$P_b = 4\pi^3 \rho e^{-kd_s} s_{3,max} V_s H / T^3$$

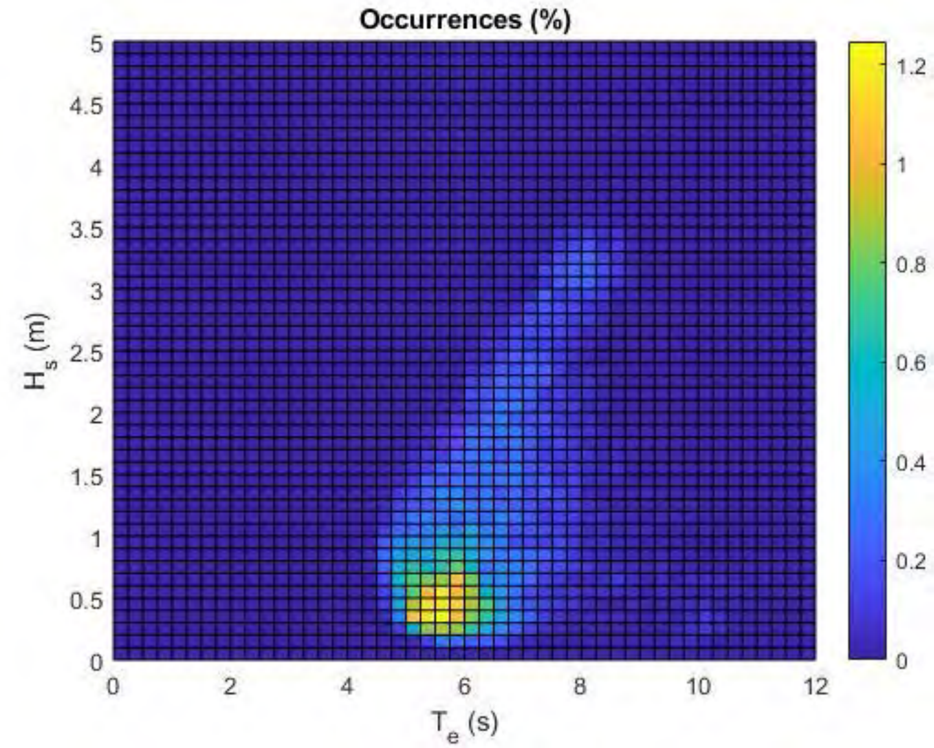


Design process

Methodology

Island of Pantelleria

Wave scatter



Sea site → *Design wave* → *Budal diagram* (← *Power limits*) → *Physical volume* / *PTO stroke* → *Numerical modelling* (← *Control*) → *Optimization* → *Experiments*

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Design process

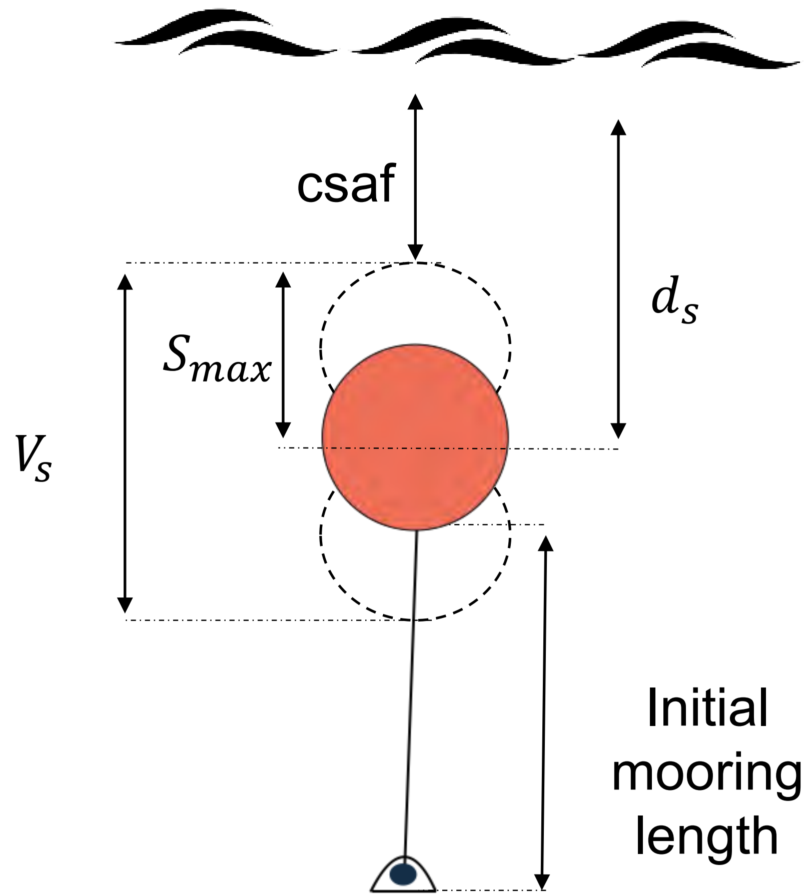
$$V_s = b \cdot V$$

$$V_s = 2 \cdot \pi \cdot r^2 \cdot S_{max}$$

$$S_{max} = b \cdot \frac{2}{3} \cdot r$$

$$d_s = c \cdot \left(r + b \cdot \frac{2}{3} \cdot r \right)$$

$$csaf = d_s - (r + s_{max})$$



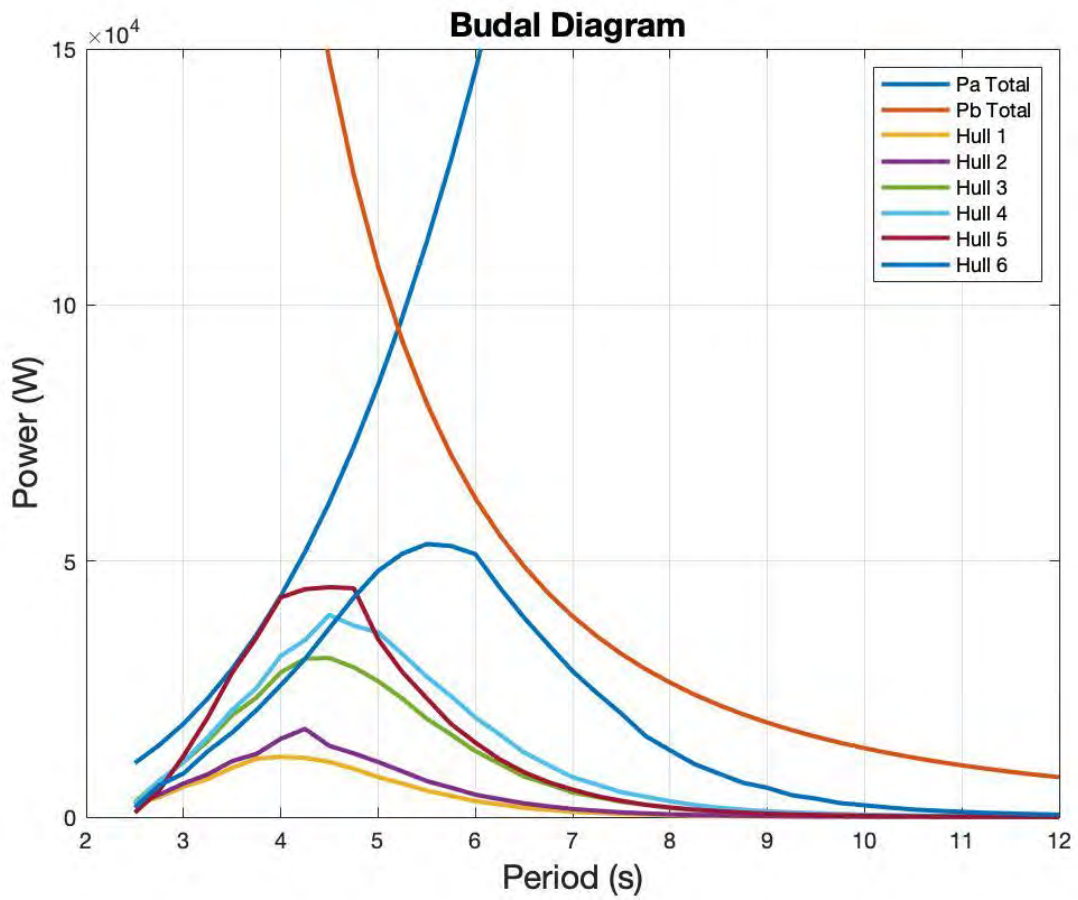
Design process

Device characteristics

TECHNICAL DETAILS OF THE SIX DIFFERENT HULLS

Symbol	Hull 1	Hull 2	Hull 3	Hull 4	Hull 5	Hull 6
m (kg)	56085	66088	126045	169851	246565	404449
R (m)	2.44	2.58	3.2	3.53	4	4.71
V (m^3)	60.8	71.64	136.63	184.12	267.13	438.44
Vs(m^3)	60.8	64.48	81.98	92.06	107	131.53
c	1.1	1.1	1.1	1.1	1.1	1.1
csaf	0.41	0.41	0.45	0.47	0.51	0.57
b	1	0.9	0.6	0.5	0.4	0.3
ds (m)	4.47	4.53	4.92	5.18	5.57	6.22
s3max (m)	1.63	1.55	1.28	1.18	1.07	0.94

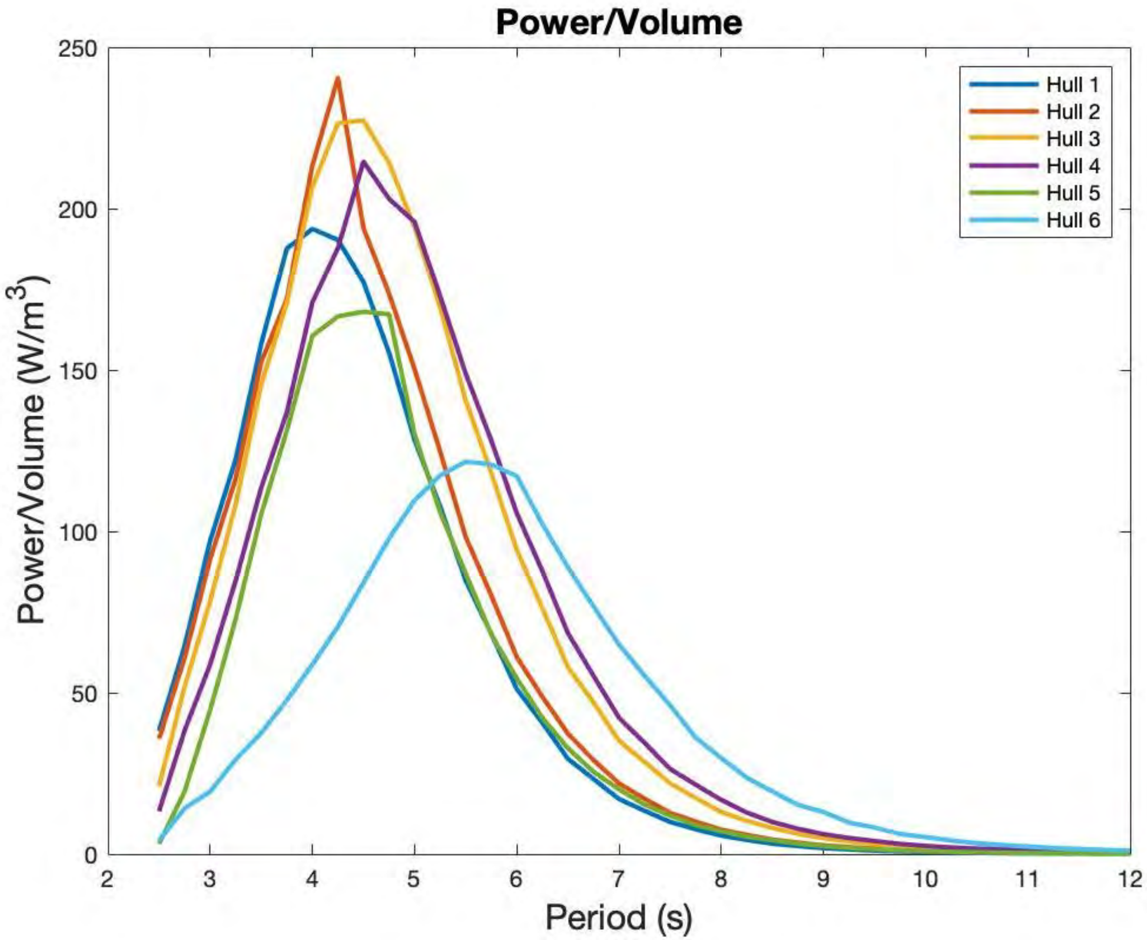
Budal diagram



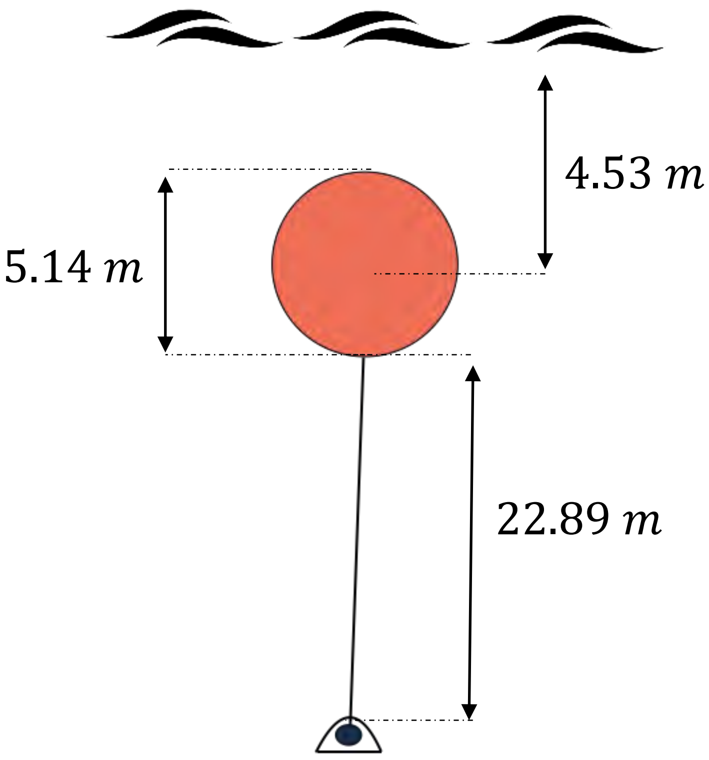
Design process

Budal Diagram

Power per volume performance



Hull 2 details

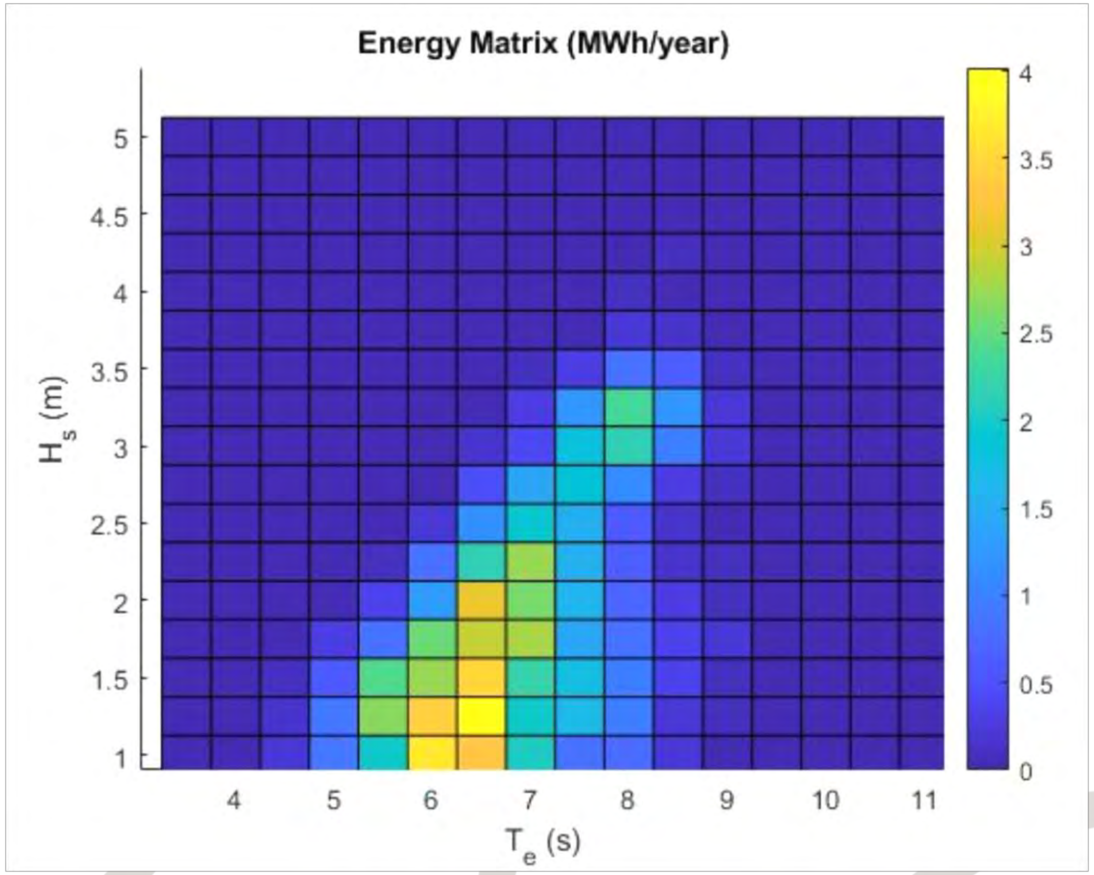


$J_T = 5{,}048\ kW/m$

Design process

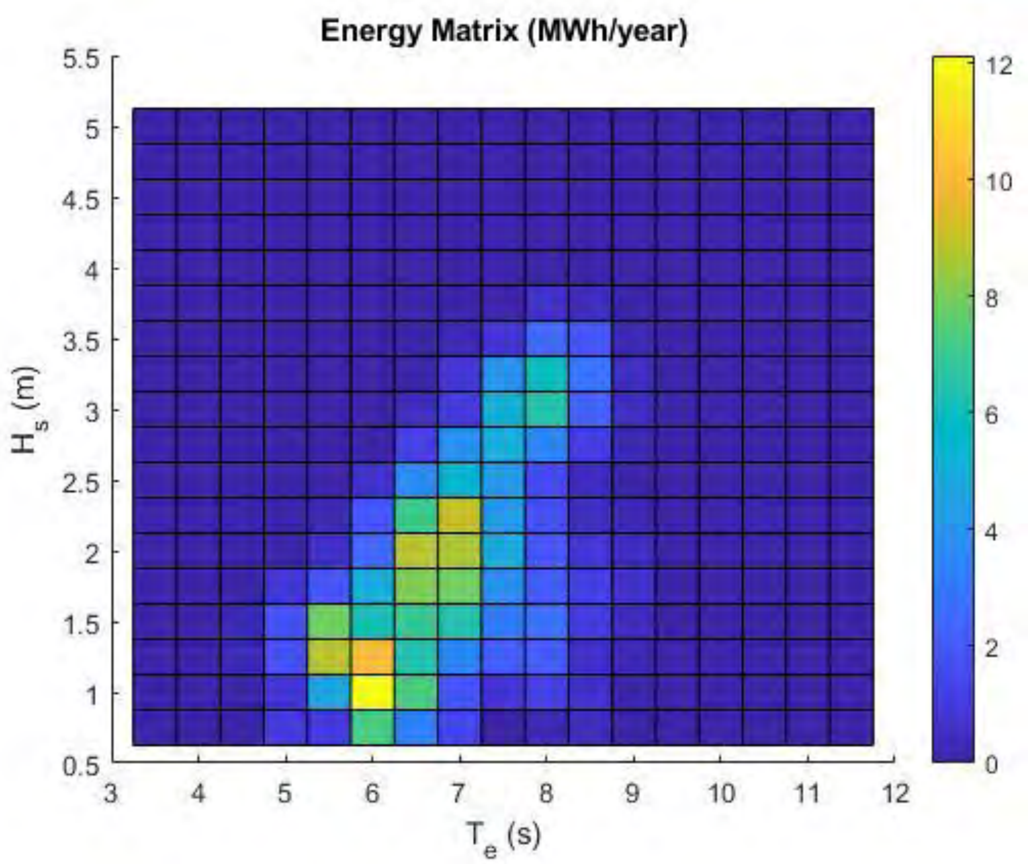
Performance

Device 25 kW



Annual production: 108.3374 MWh

Device 50 kW

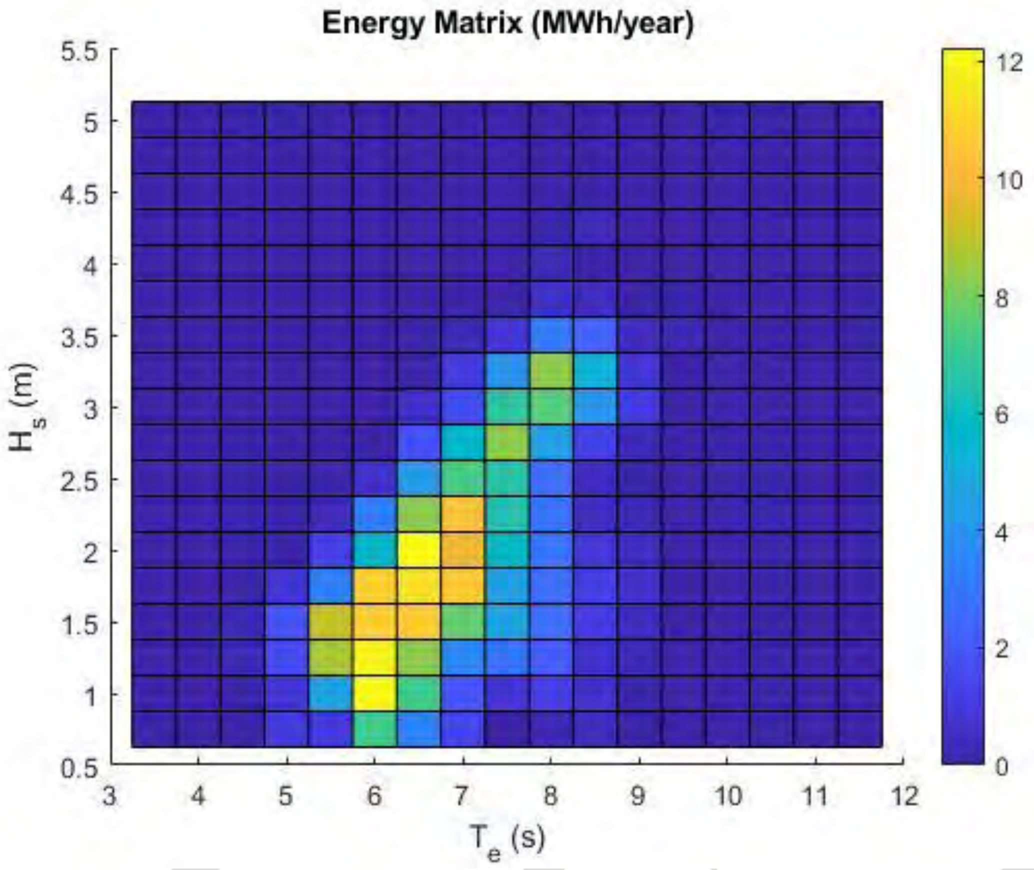


Annual production: 274.1678 MWh

Design process

Performance

Device 75 kW



Annual production: 346.5451 MWh

ANNUAL ENERGY PRODUCTION

Power Capacity	Annual Production	Capacity Factor
25 kW	108.3374 MWh	49.32%
50 kW	274.1678 MWh	62.56%
75 kW	346.5451 MWh	52.66%
100 kW	349.6523 MWh	39.84%

$$\text{Capacity factor} = \frac{\text{Actual energy generated}}{\text{Capacity} \times \text{Time}}$$

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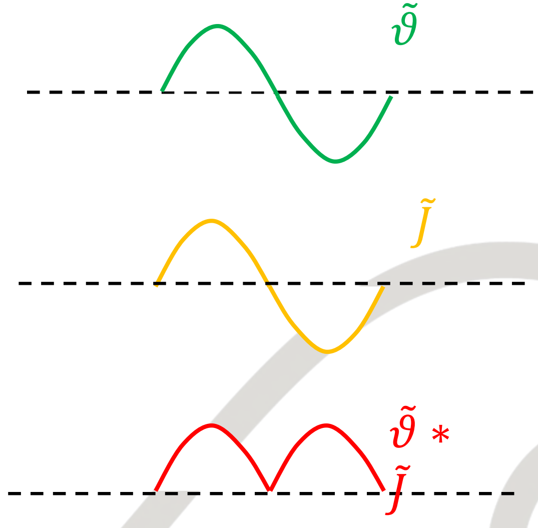
Extremum seeking control

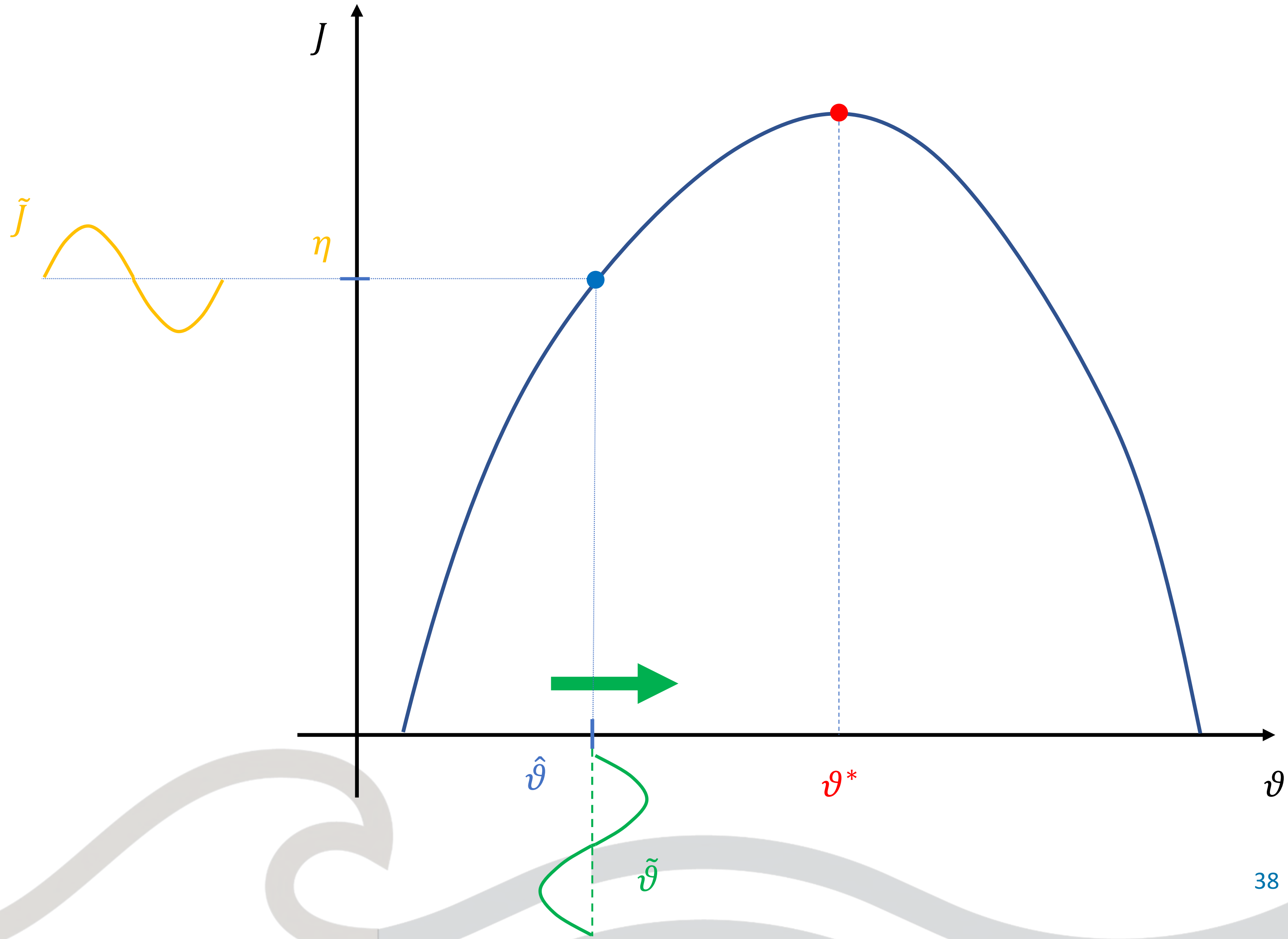
Description
Results

Extremum Seeking Control

Case 1: Positive gradient

The signal $\tilde{\vartheta} * \tilde{J}$ obtained is non-negative.

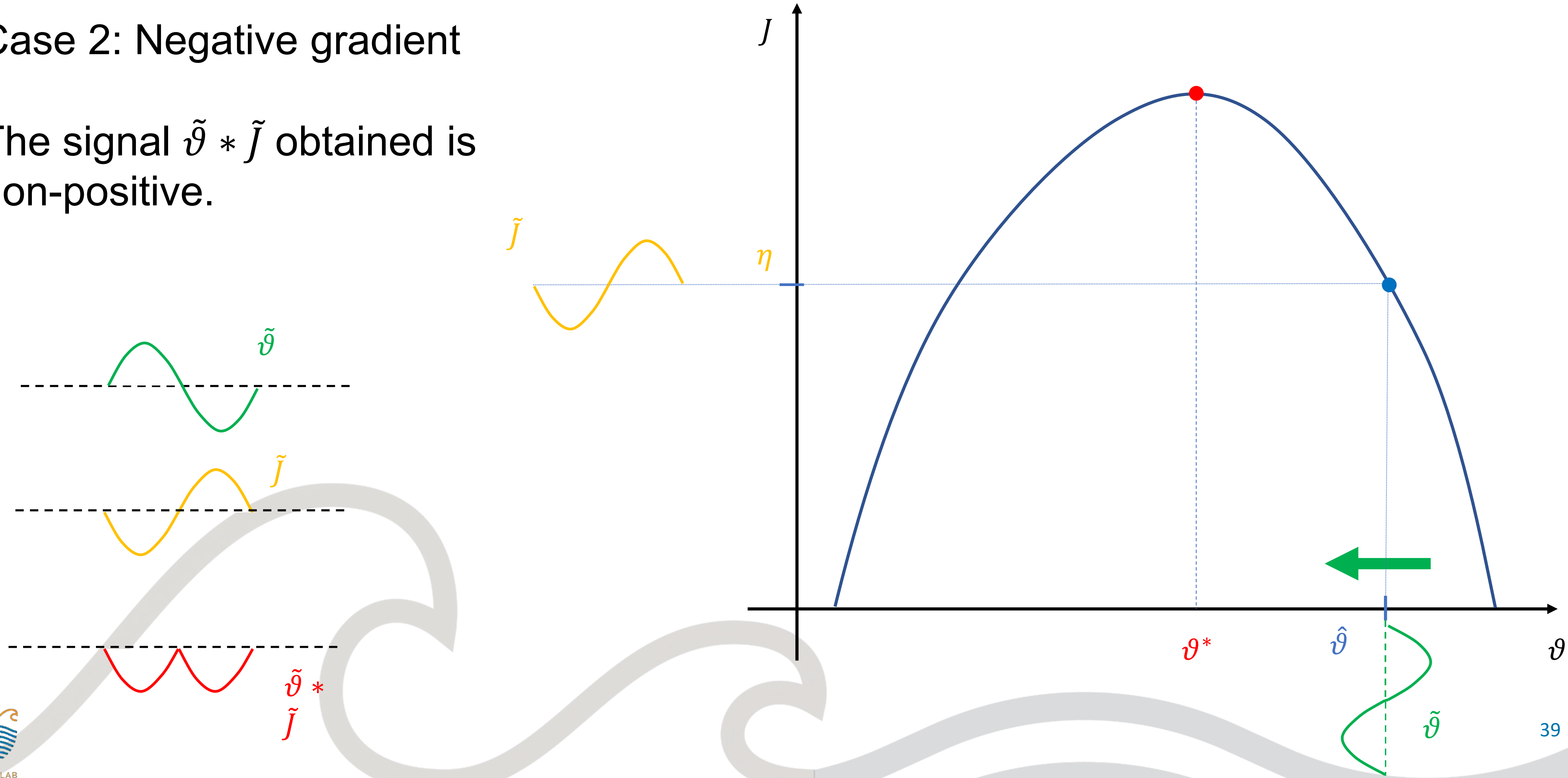




Extremum Seeking Control

Case 2: Negative gradient

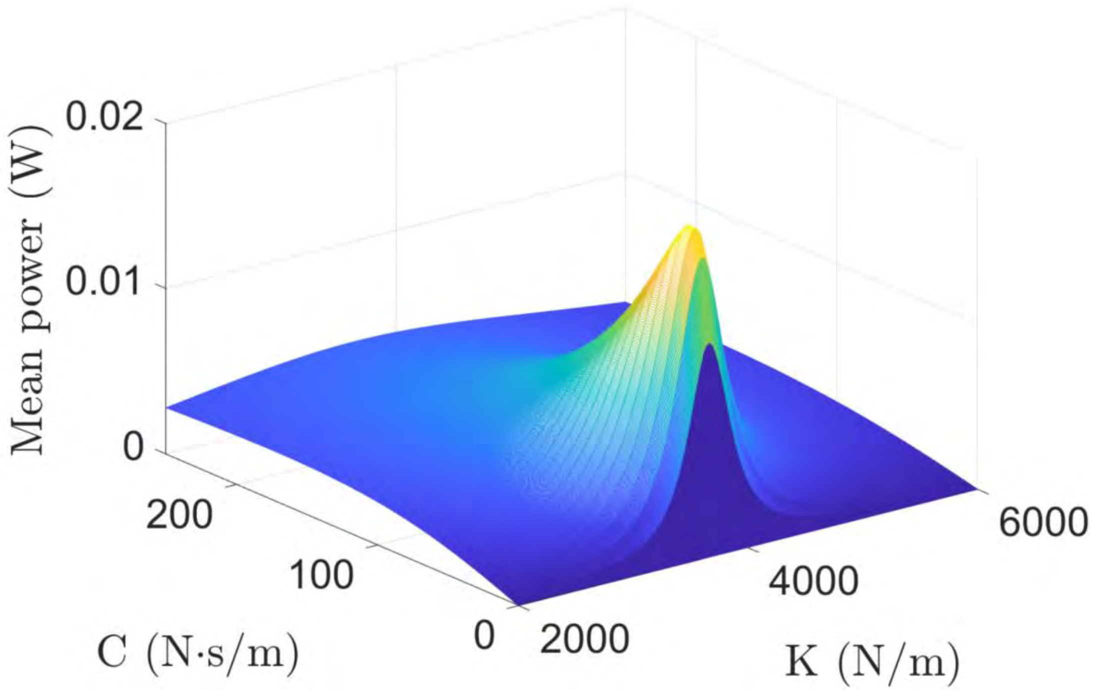
The signal $\tilde{\vartheta} * \tilde{J}$ obtained is non-positive.



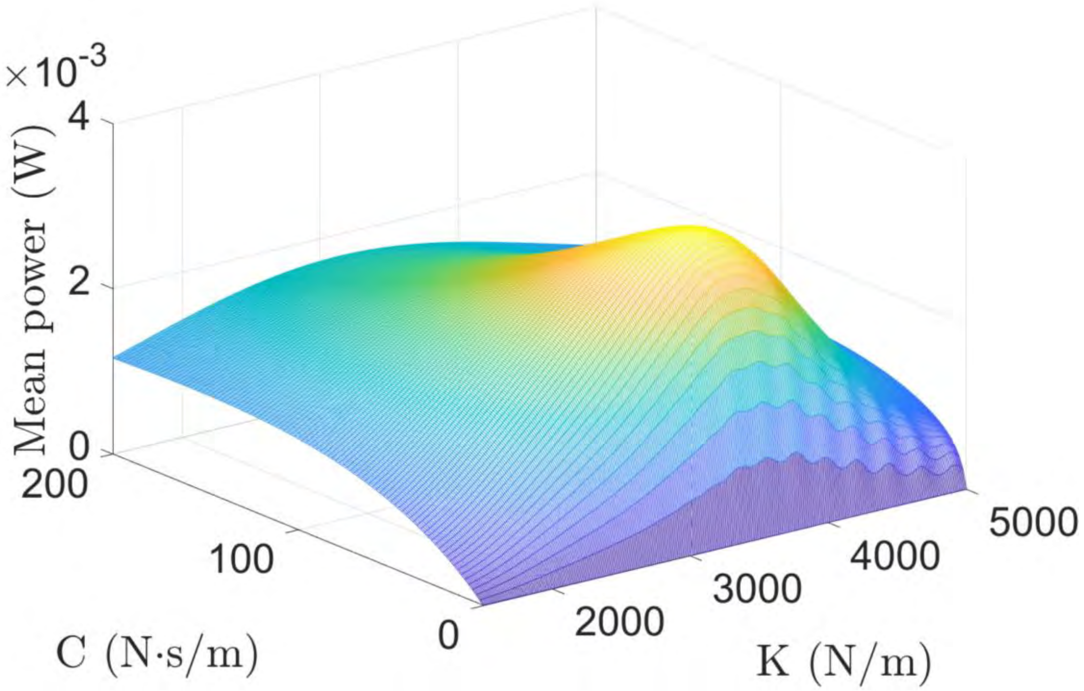
Extremum Seeking Control

Description

Regular waves



Irregular waves



Power vs. PTO coefficients reference-to-output map for a cylinder subject to regular waves of period $T = 0{,}625$ s and height two-dimensional $H = 0{,}01$ m. The optimal PTO coefficients are: $K_{\text{opt}} = 3720$ N/m and $C_{\text{opt}} = 18$ N·s/m

Power vs. PTO coefficients reference-to-output map for a two-dimensional cylinder subject to irregular waves obtained through a JONSWAP spectrum of peak-period $T_p = 0{,}625$ s and significant height $H_s = 0{,}01$ m. The optimal PTO coefficients are: $K_{\text{opt}} = 3440$ N/m and $C_{\text{opt}} = 32$ N·s/m

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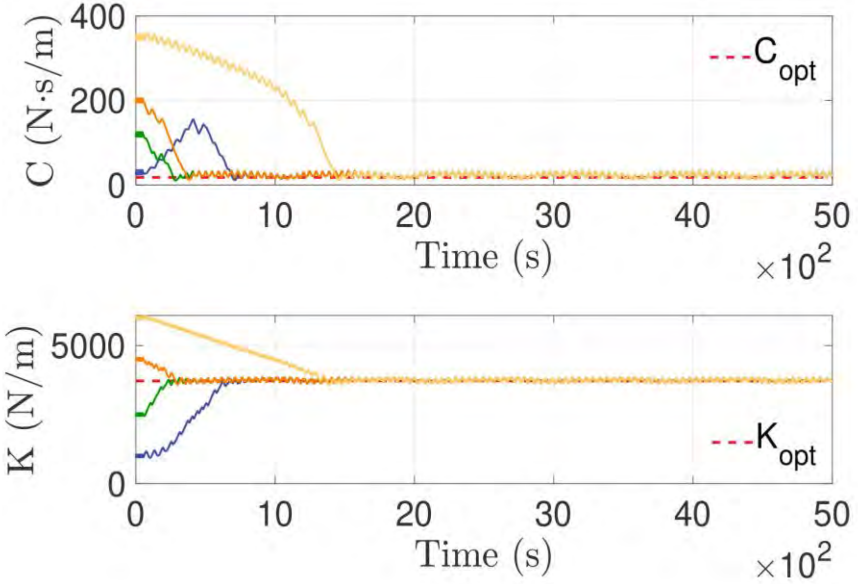
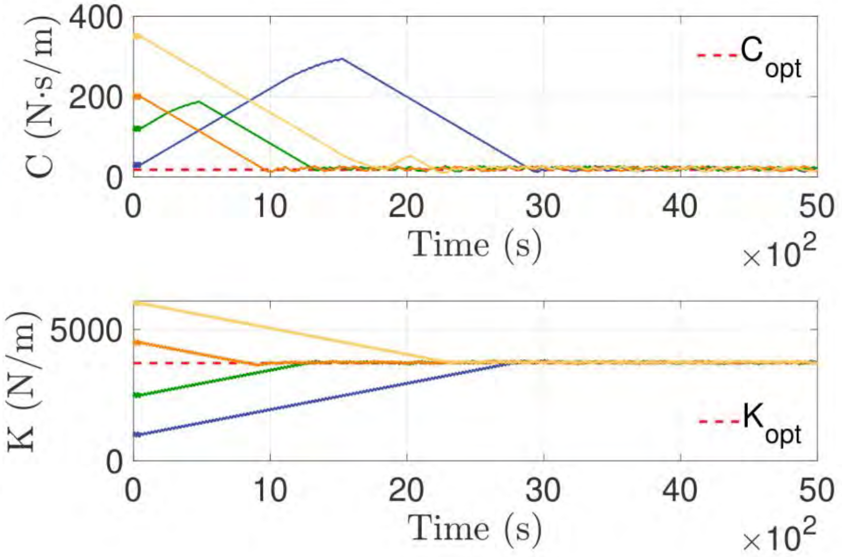
Extremum Seeking Control

Results

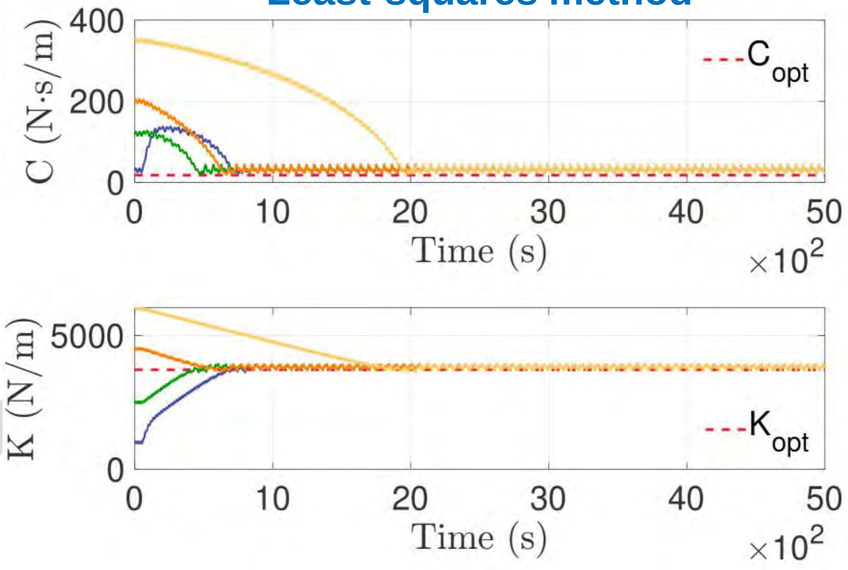
Sliding-mode ES



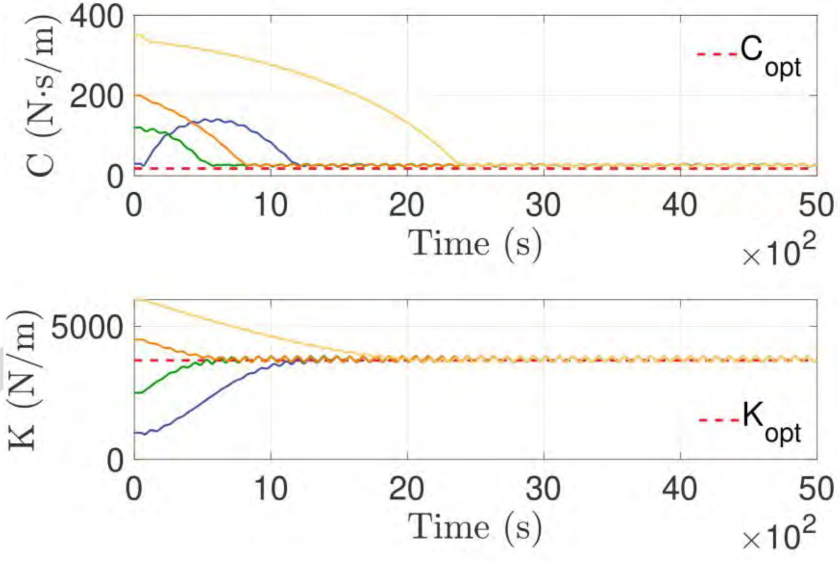
Relay ES



Least-squares method



Perturbation-based ES



Regular waves

Sea State parameters:

- $T = 0{,}625$ s
- $H = 0{,}01$ m

Optimal values for the PTO coefficients:

- $K_{\mathrm{opt}} = 3720$ N/m
- $C_{\mathrm{opt}} = 18$ N·s/m

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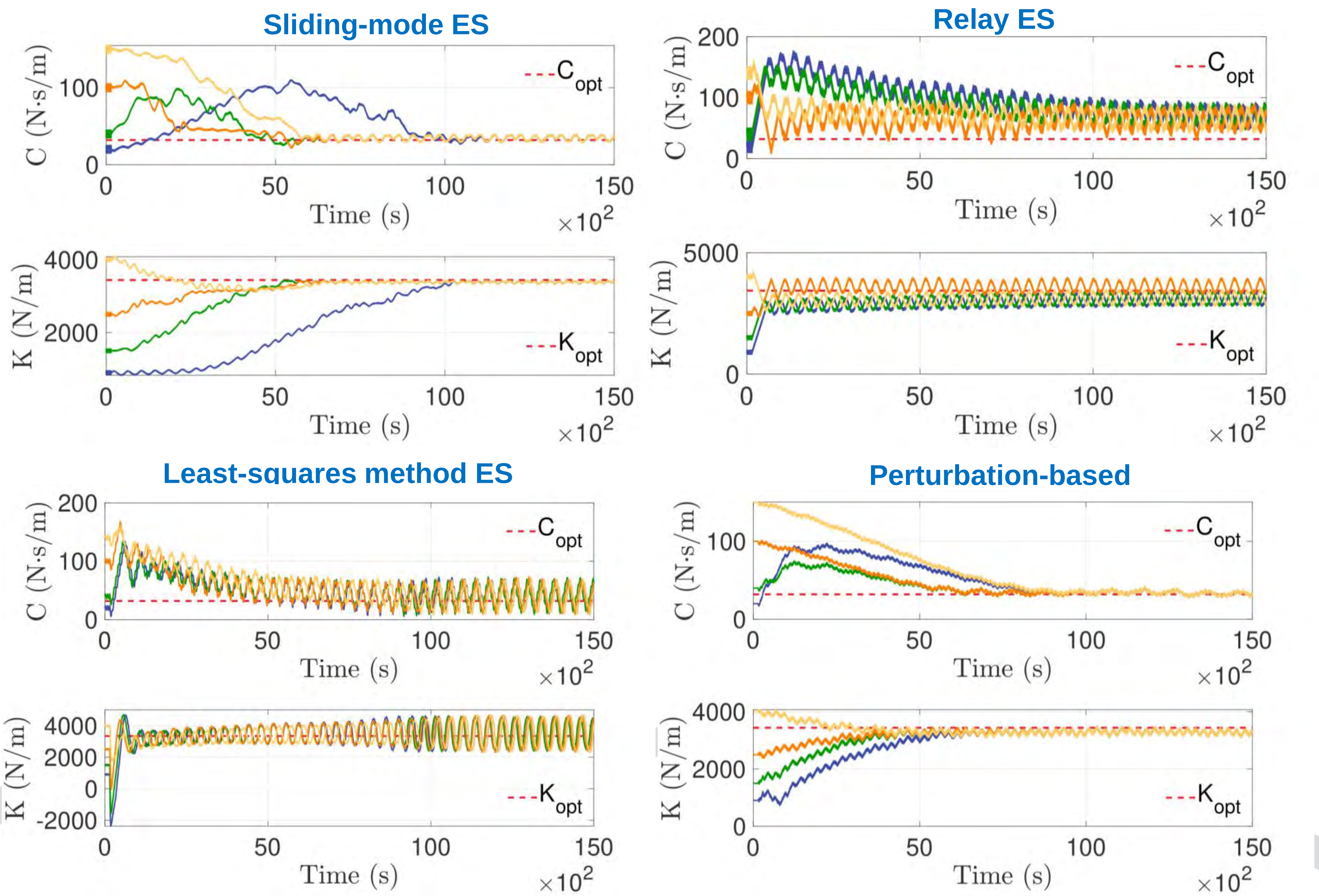
Extremum Seeking Control

Results



Irregular waves

Sea State parameters:

- $T_p = 0{,}625$ s
- $H_s = 0{,}01$ m

Optimal values for the PTO coefficients:

- $K_{\text{opt}} = 3440$ N/m
- $C_{\text{opt}} = 32$ N·s/m

Dynamic analysis of a multi-tether point absorber

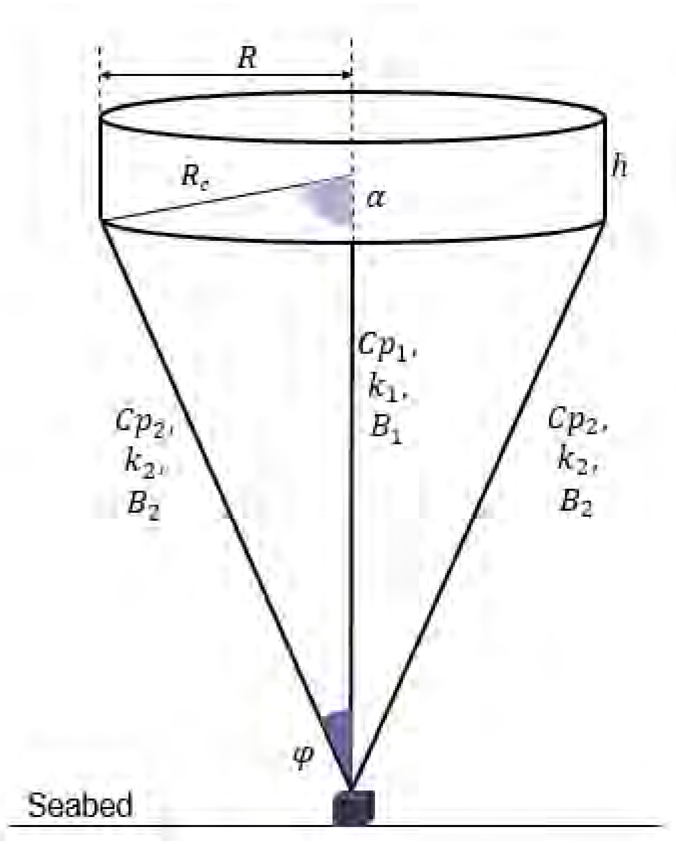
Description

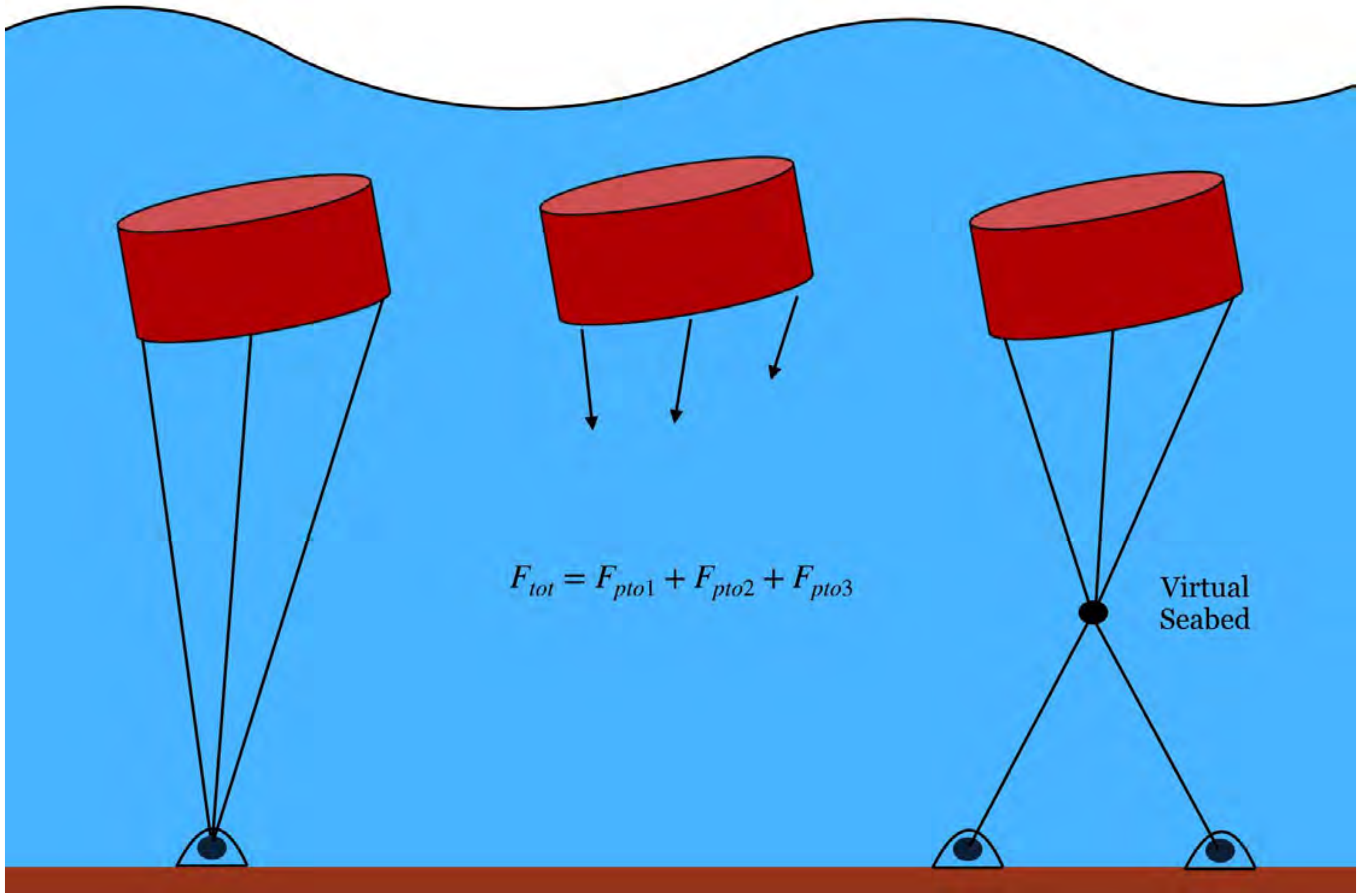
Depth control

Modal analysis

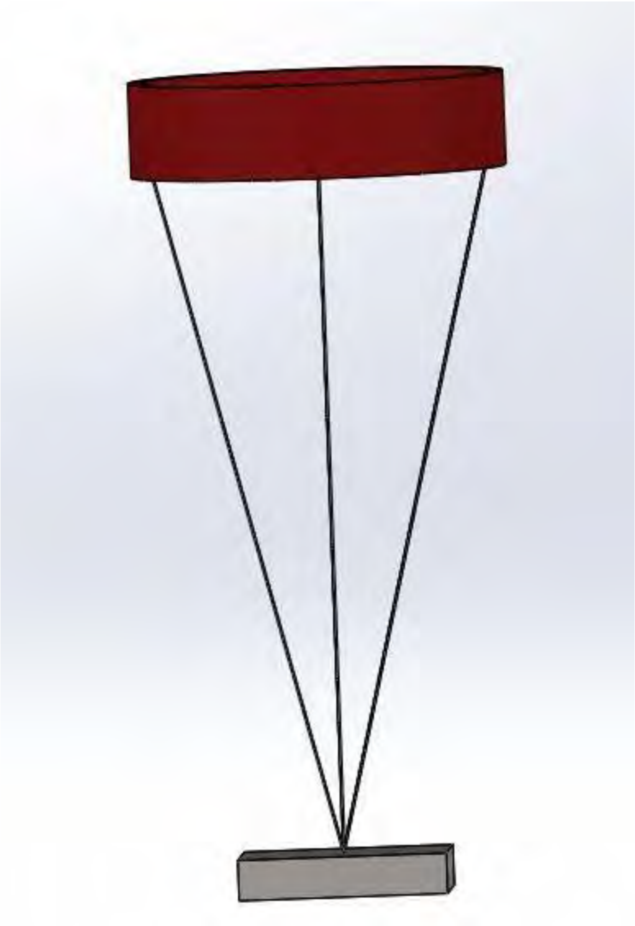
Performance results

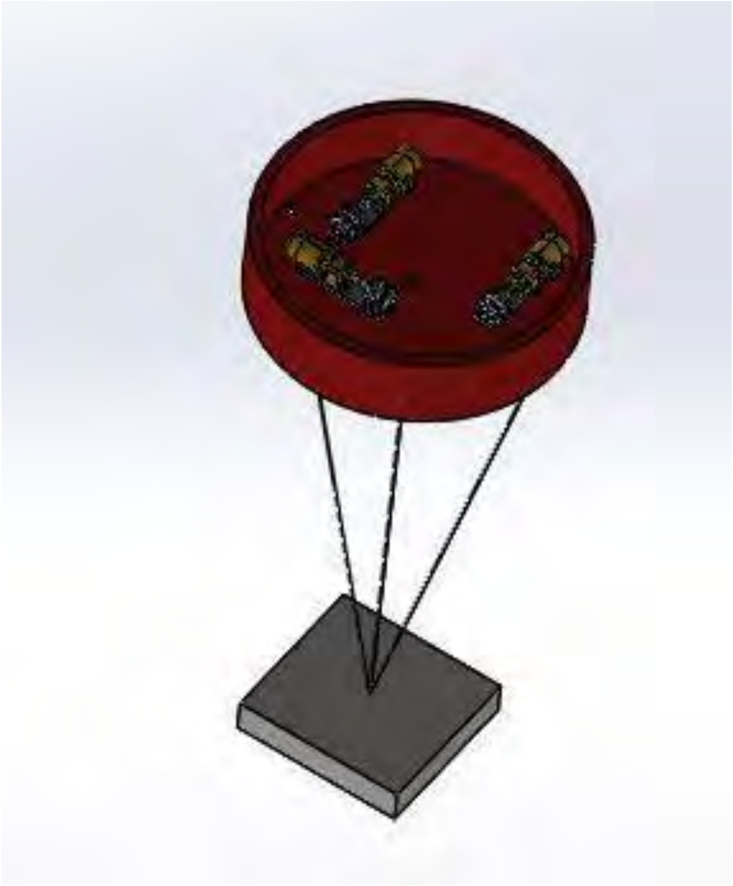
Multi – tether PA

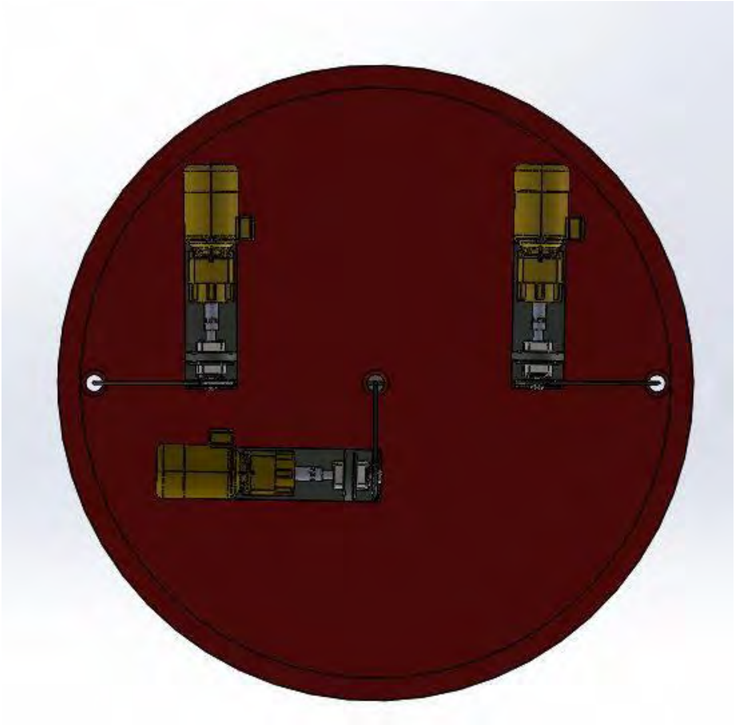




Multi – tether PA

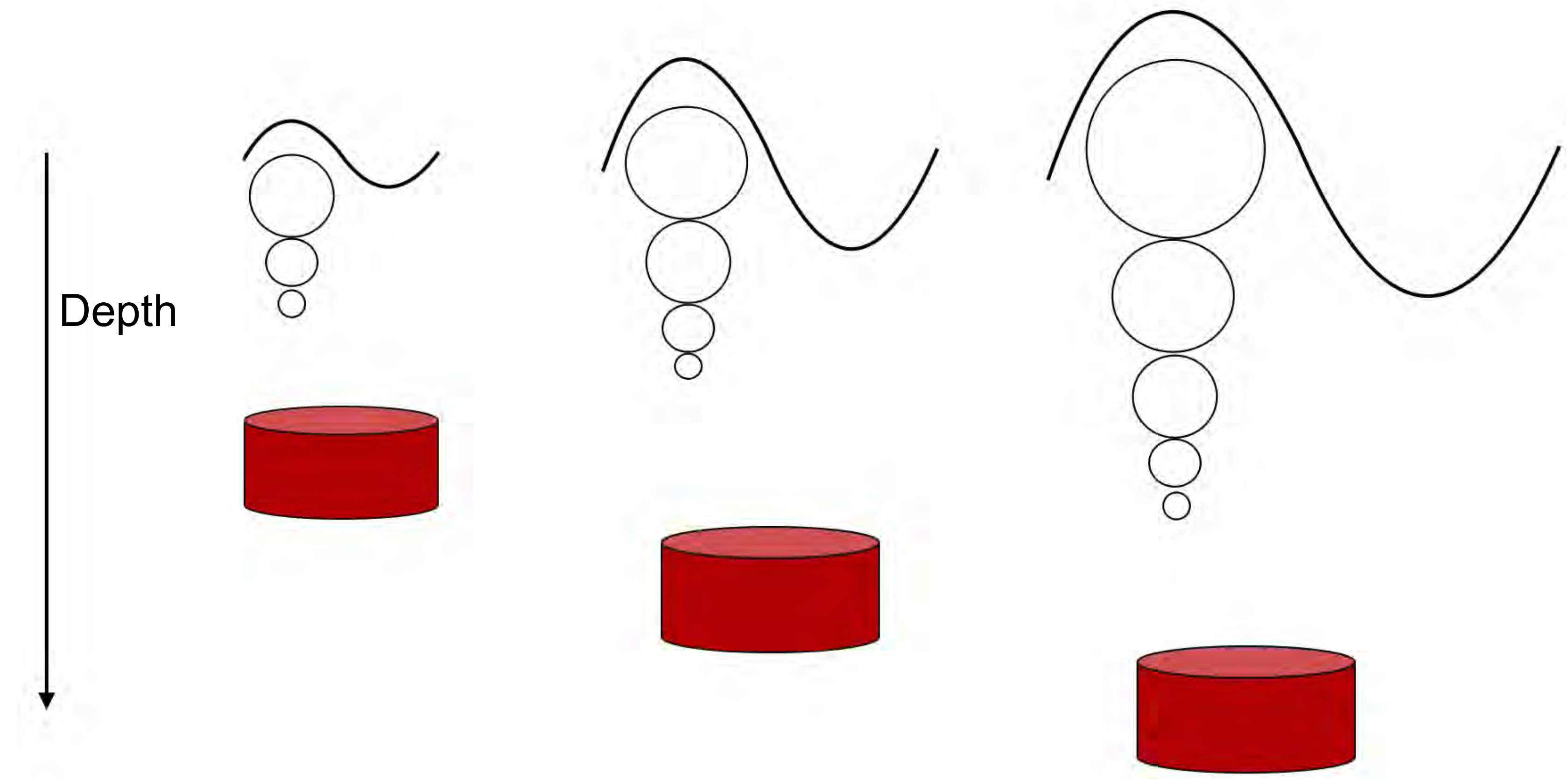


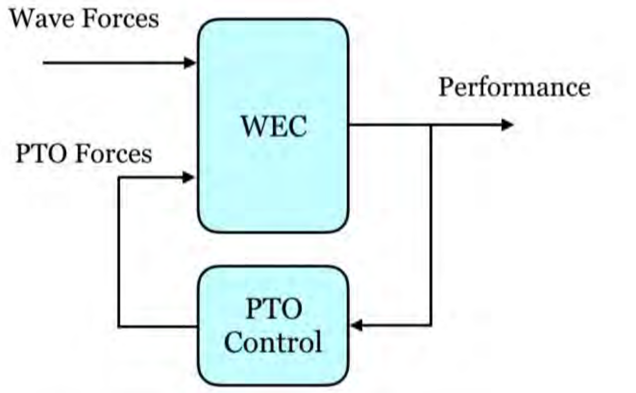


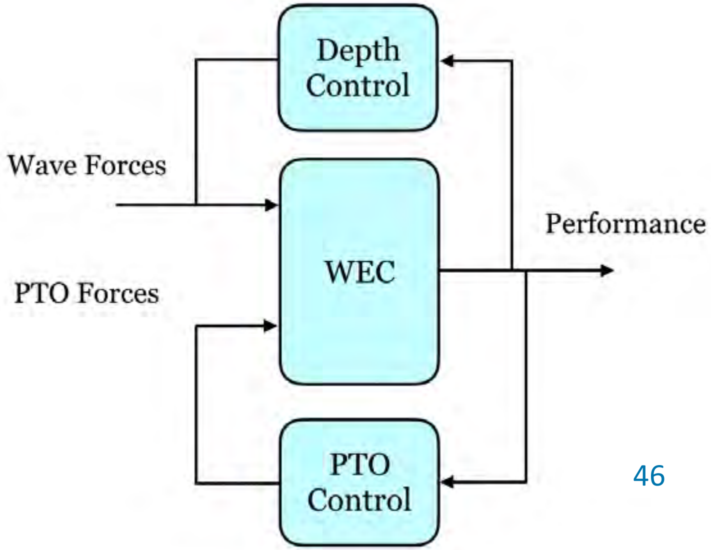


Multi – tether PA

Depth control



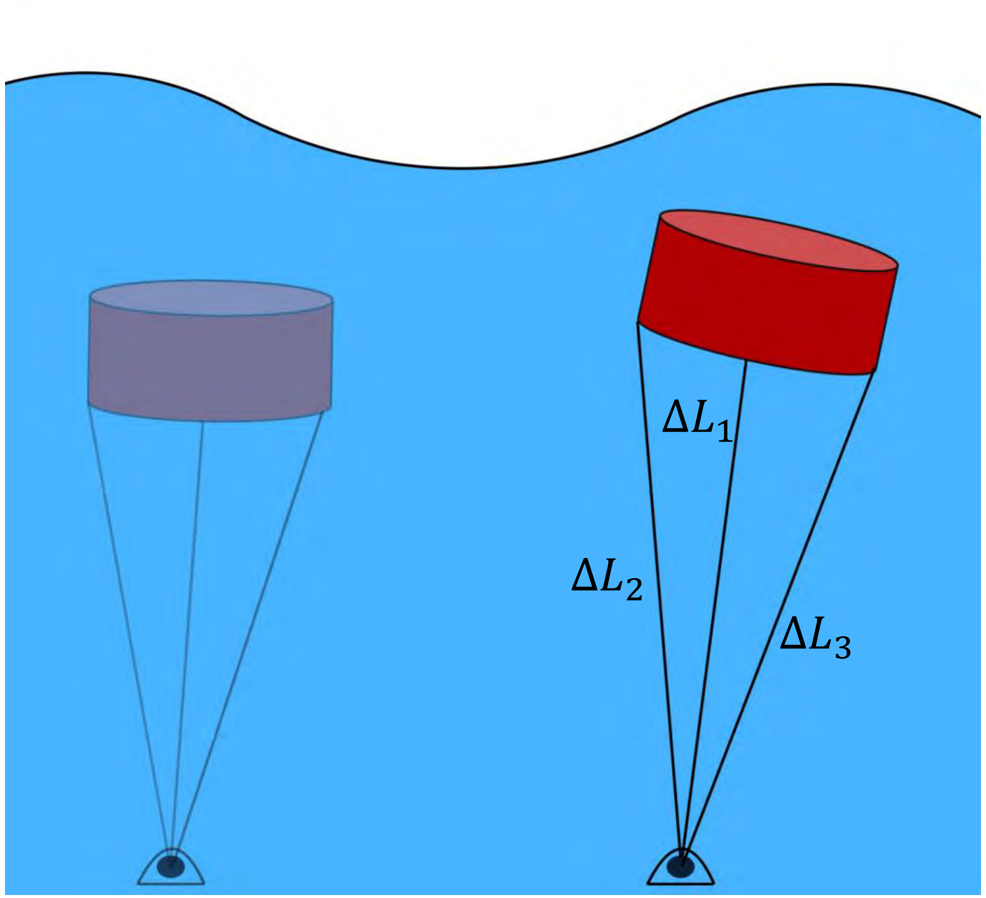




Multi – tether PA

Linearization

Linearization of the mooring dynamics using Taylor series



Tether elongation

$$\Delta L_1 = L_1 - L_1^0 = \sqrt{\left(x - \frac{h}{2}\sin\vartheta\right)^2 + \left(L_1^0 + z + \frac{h}{2}(1-\cos\vartheta)\right)^2} - L_1^0$$

$$\Delta L_2 = \sqrt{(-R_c\sin\alpha + x - R_c\vartheta\cos\alpha)^2 + (L_1^0 + z + R_c\vartheta\sin\alpha)^2} - L_2^0$$

$$\Delta L_3 = \sqrt{(R_c\sin\alpha + x - R_c\vartheta\cos\alpha)^2 + (L_1^0 + z - R_c\vartheta\sin\alpha)^2} - L_3^0$$

Stiffness matrix

$$\mathbf{K_{PTO}} = \begin{bmatrix} \begin{array}{l} \frac{Cp_1}{L_1^0} + \frac{2Cp_2}{L_2^0}\cos^2\varphi \\ + 2K_2\sin^2\varphi \end{array} & 0 & \begin{array}{l} -\frac{Cp_1 h}{2L_1^0} - \frac{2Cp_2 R_c\cos\alpha}{L_2^0} \\ + \frac{2Cp_2 R_c\sin\varphi\sin(\alpha+\varphi)}{L_2^0} \\ - 2K_2 R_c\sin\varphi\sin(\alpha+\varphi) \end{array} \\ 0 & \begin{array}{l} \frac{2Cp_2\sin^2\varphi}{L_2^0} + K_1 \\ + 2K_2\cos^2\varphi \end{array} & 0 \\ \begin{array}{l} -\frac{Cp_1 h}{2L_1^0} - \frac{2Cp_2 R_c\cos\alpha}{L_2^0} \\ + \frac{2Cp_2 R_c\sin\varphi\sin(\alpha+\varphi)}{L_2^0} \\ - 2K_2 R_c\sin\varphi\sin(\alpha+\varphi) \end{array} & 0 & \begin{array}{l} \frac{Cp_1}{L_1^0}\left(\frac{h}{2}\right)^2 + \frac{Cp_1 h}{2} \\ + \frac{2Cp_2 R_c^2}{L_2^0} - \frac{2Cp_2\left(R_c\sin(\alpha+\varphi)\right)^2}{L_2^0} \\ + 2Cp_2 R_c\cos(\alpha+\varphi) \\ + 2K_2\left(R_c\sin(\alpha+\varphi)\right)^2 \end{array} \end{bmatrix}$$

Multi – tether PA

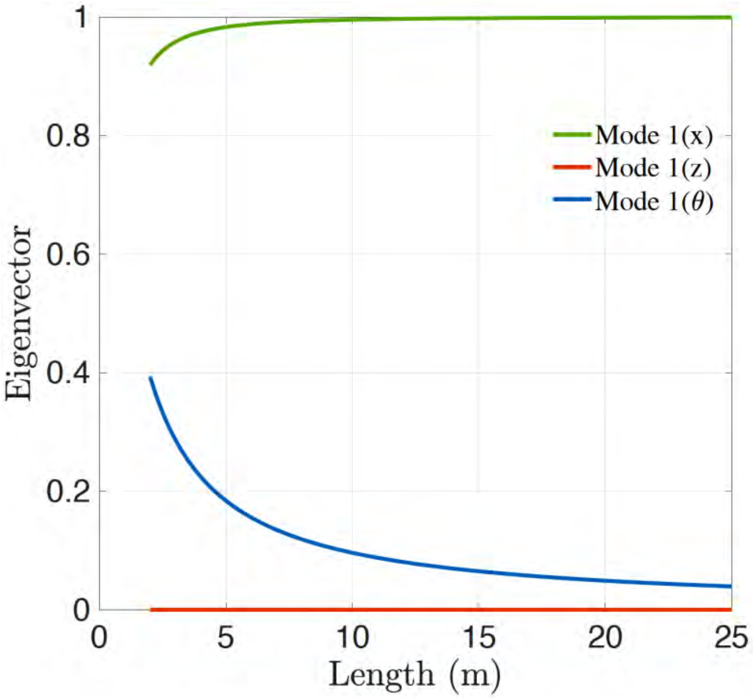
Modal analysis

$$[(\boldsymbol{M} + \boldsymbol{A}(\omega))^{-1}\boldsymbol{K}_{PTO} - \lambda \boldsymbol{I}]\boldsymbol{v} = 0$$

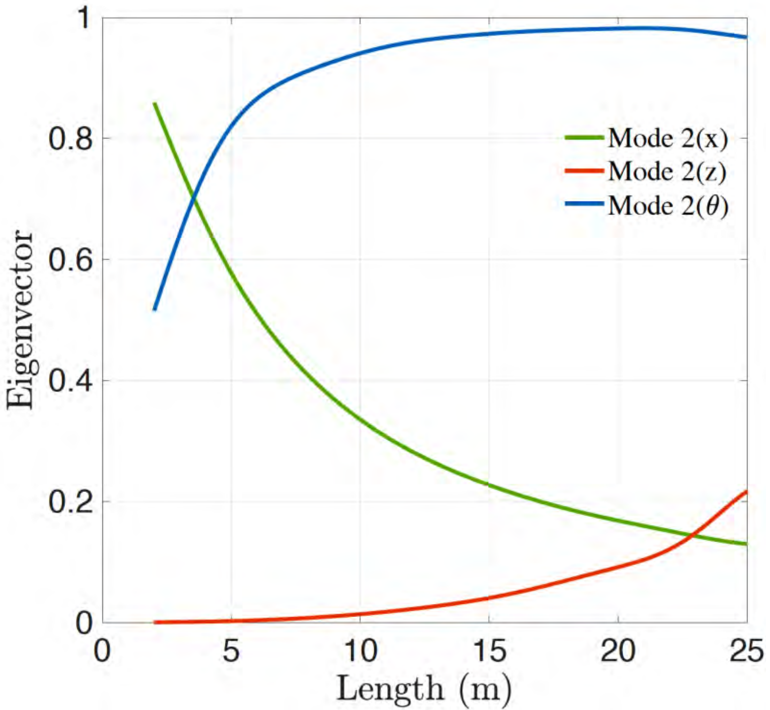
Eigenvalue $\boldsymbol{\lambda_i = \omega_i^2}$

Eigenvector $\boldsymbol{V = [v_1 \quad v_2 \quad v_3]}$

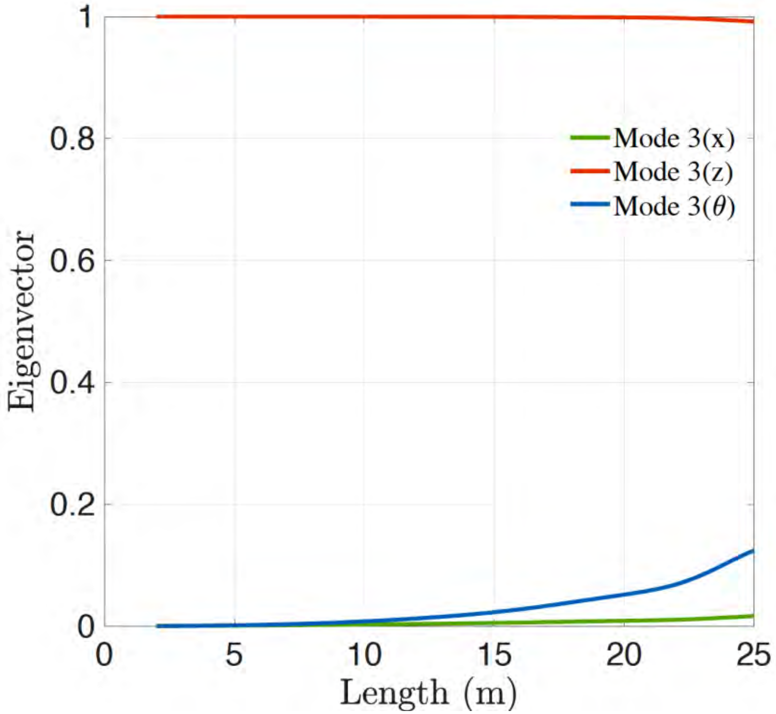
Mode 1



Mode 2

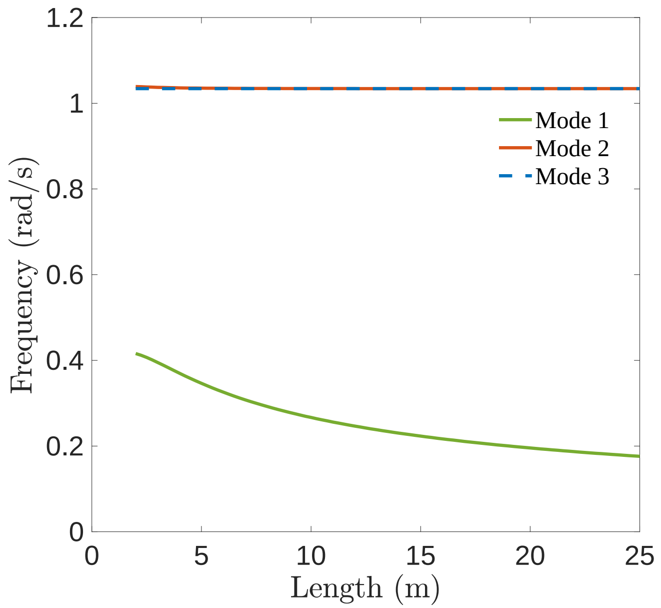


Mode 3



Multi – tether PA

Modal analysis



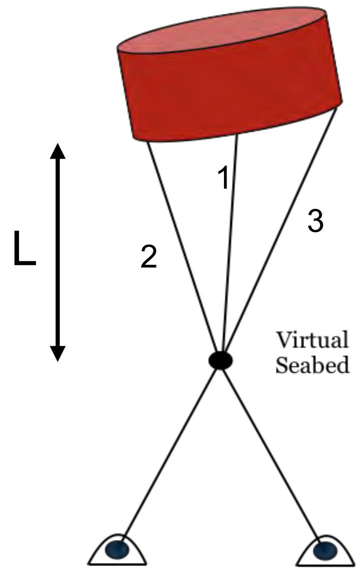
Characteristics

Wave Height = 0.1 m

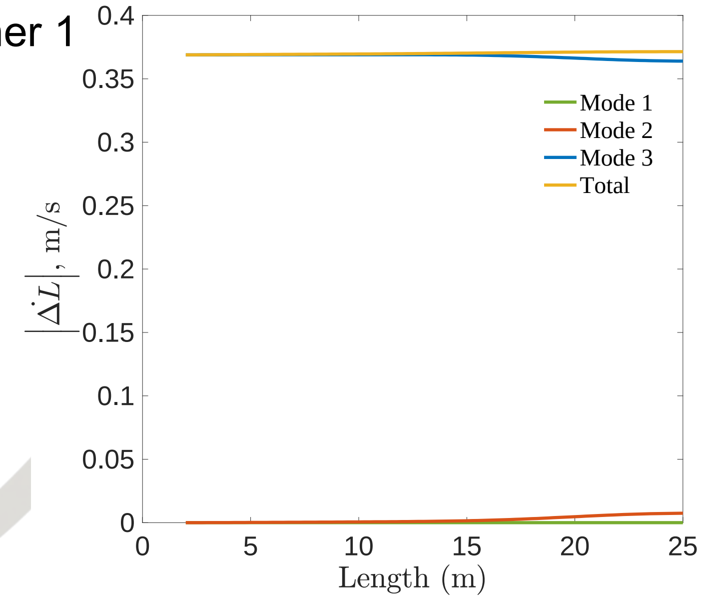
Radius = 3.25 m

Height = 2.16 m

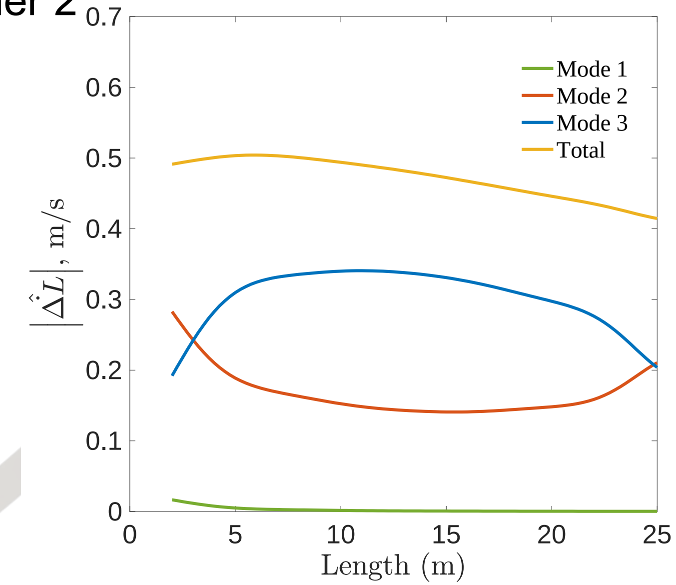
Depth = 30 m



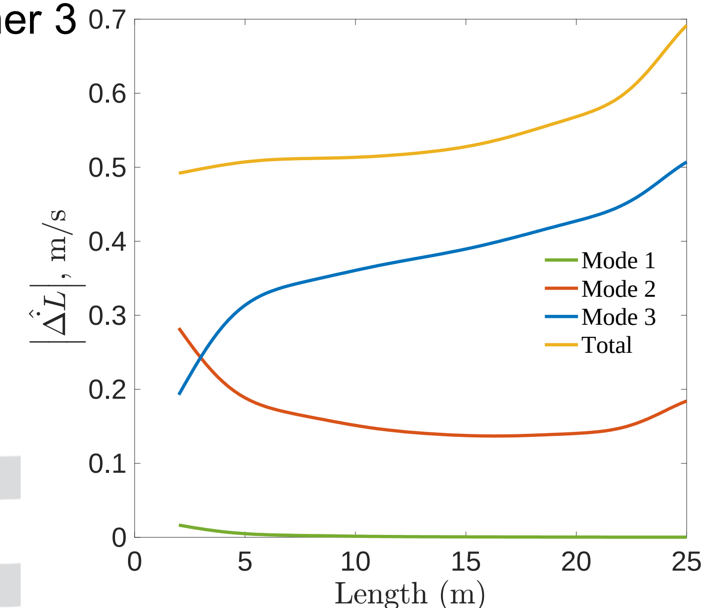
Tether 1



Tether 2

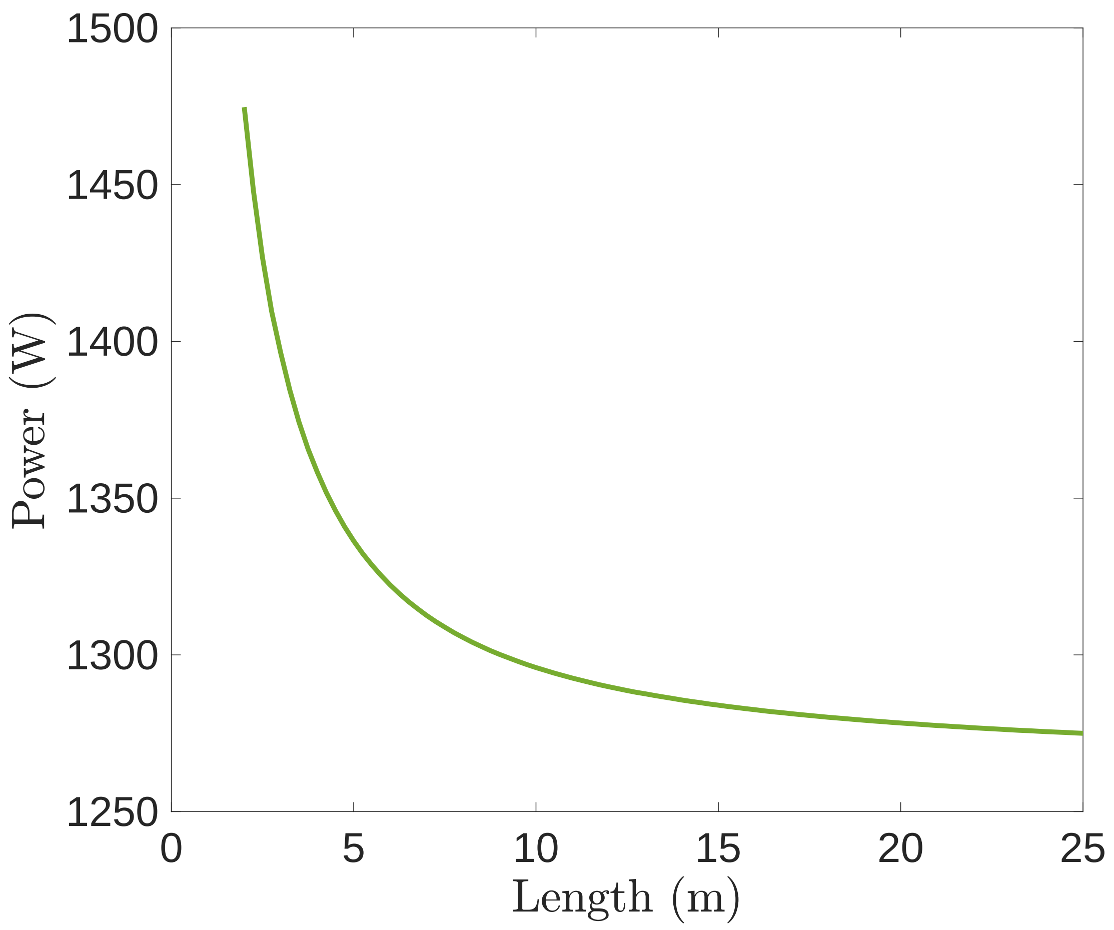


Tether 3



Multi – tether PA

Performance



$$P_{tot} = \sum_i \frac{1}{2} B_{PTO,i} \left|\widehat{\Delta \dot{L}_i}\right|^2$$

$$P_{tot} = \underbrace{B_2 \sin^2 \varphi \, \hat{\dot{x}} \widehat{\dot{x}^*}}_{P_{surge}} + \underbrace{\left(\frac{B_1}{2} + B_2 \cos^2 \varphi\right) \hat{\dot{z}} \widehat{\dot{z}^*}}_{P_{heave}}$$

$$+ \underbrace{B_2 (R_c \sin(\alpha + \varphi))^2 \hat{\dot{\vartheta}} \widehat{\dot{\vartheta}^*}}_{P_{pitch}}$$

$$- \underbrace{B_2 R_c \sin \varphi \sin(\alpha + \varphi) \left(\hat{\dot{x}} \widehat{\dot{\vartheta}^*} + \widehat{\dot{x}^*} \hat{\dot{\vartheta}}\right)}_{P_{cross}}$$

Dynamic analysis of an interconnected WEC array

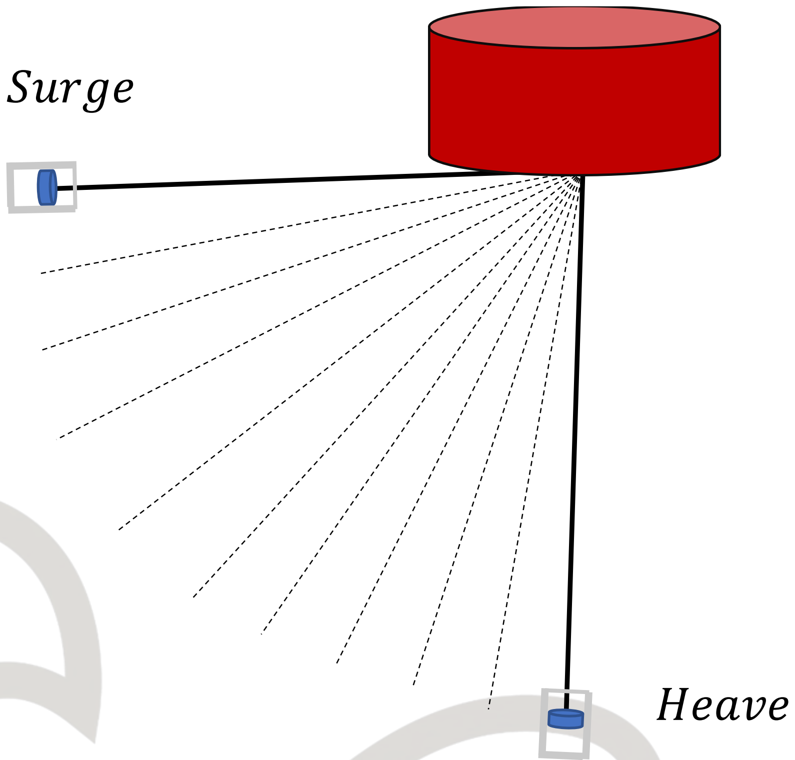
Description

Results

MORE LAB

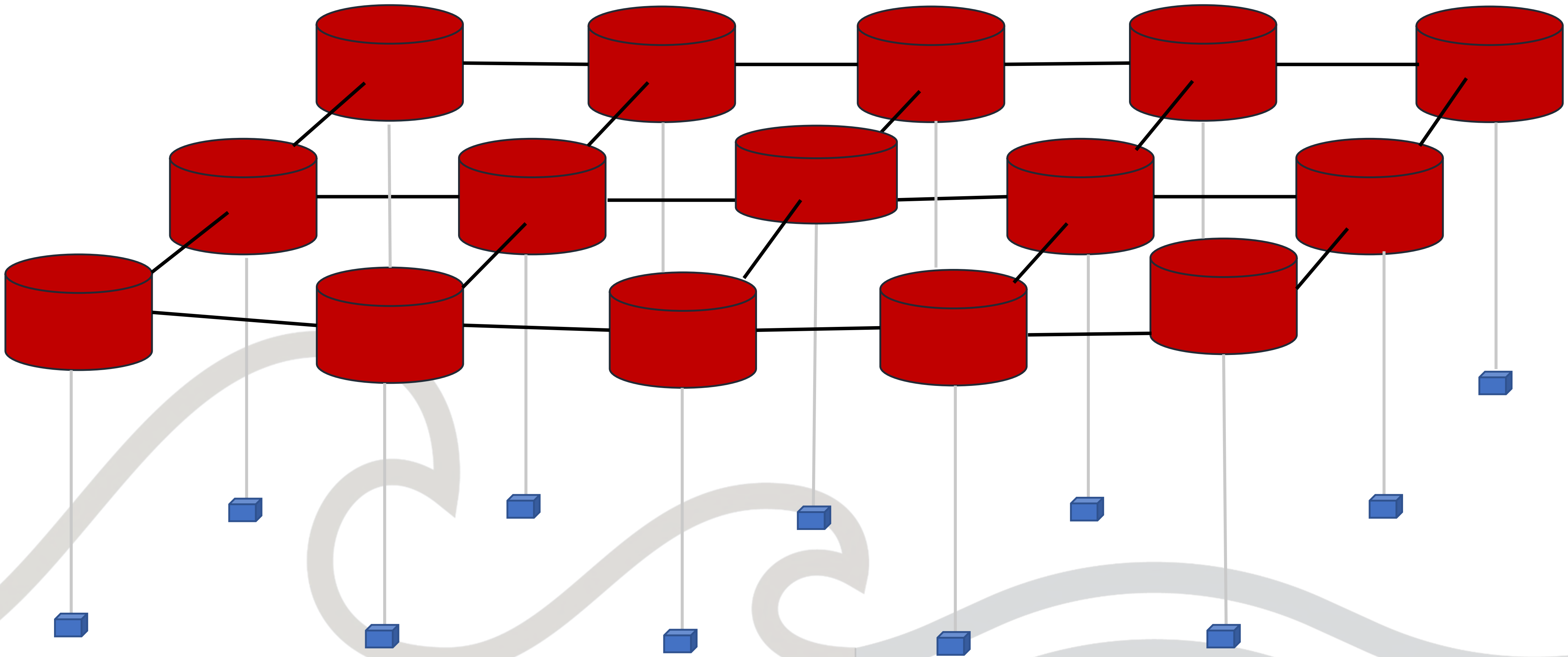
Interconnected array

Description



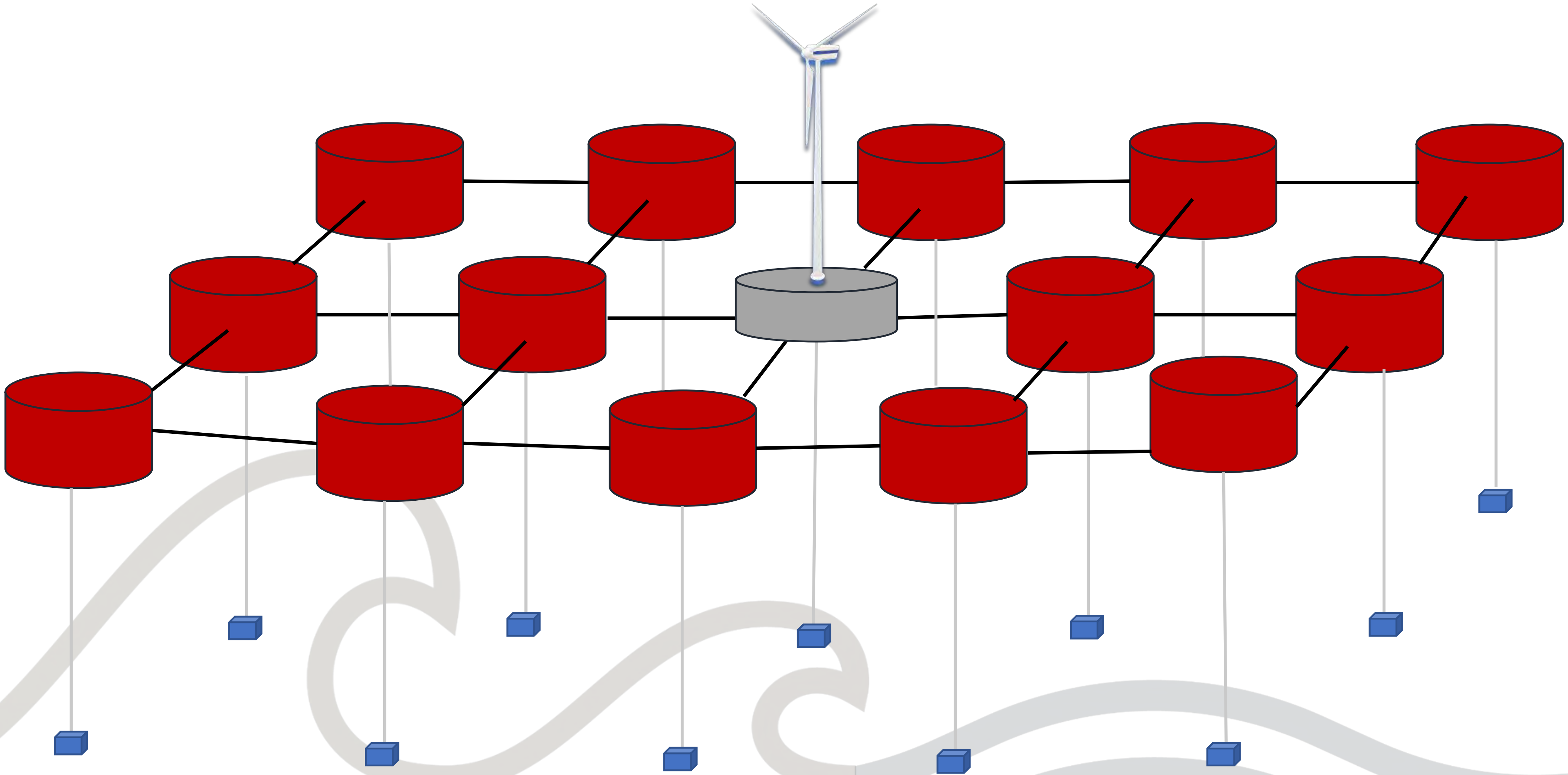
Interconnected array

Combined with offshore wind

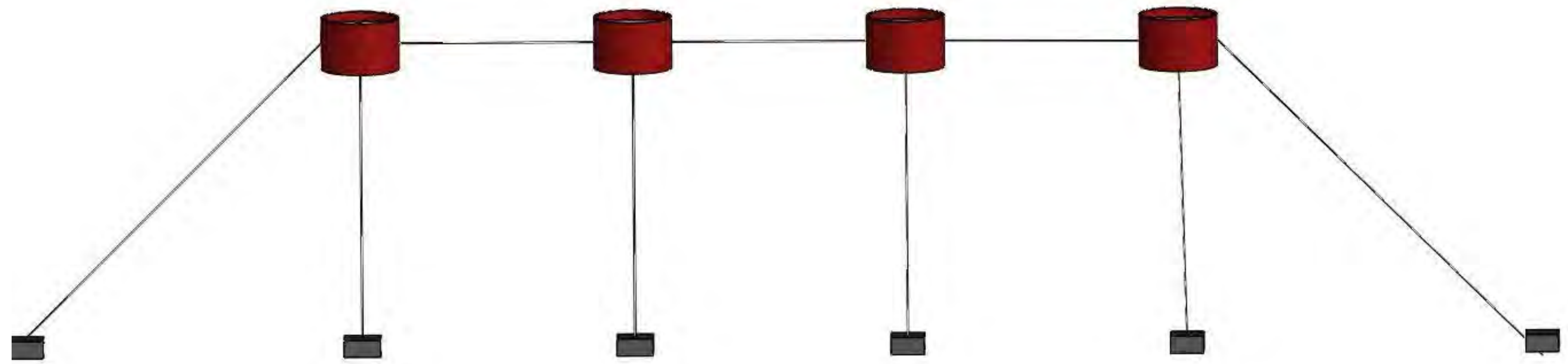


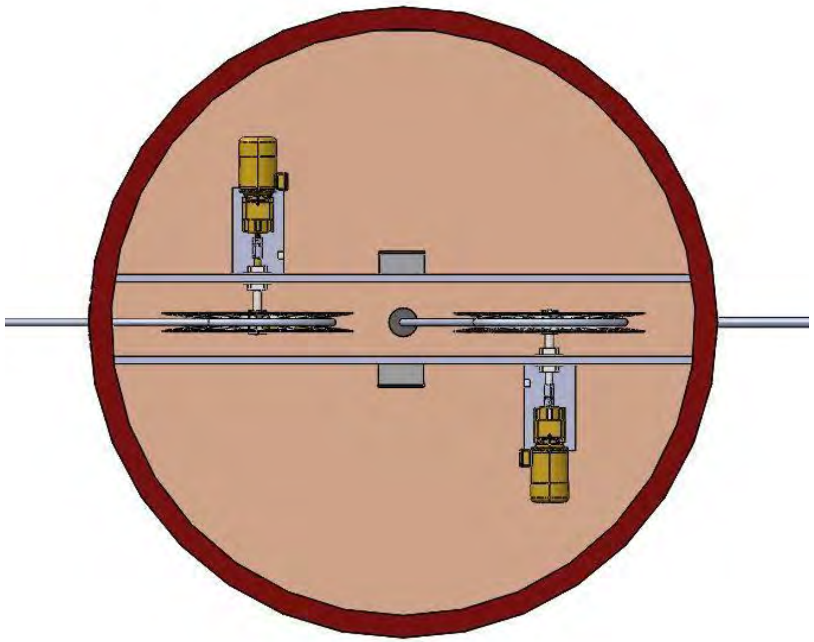
Interconnected array

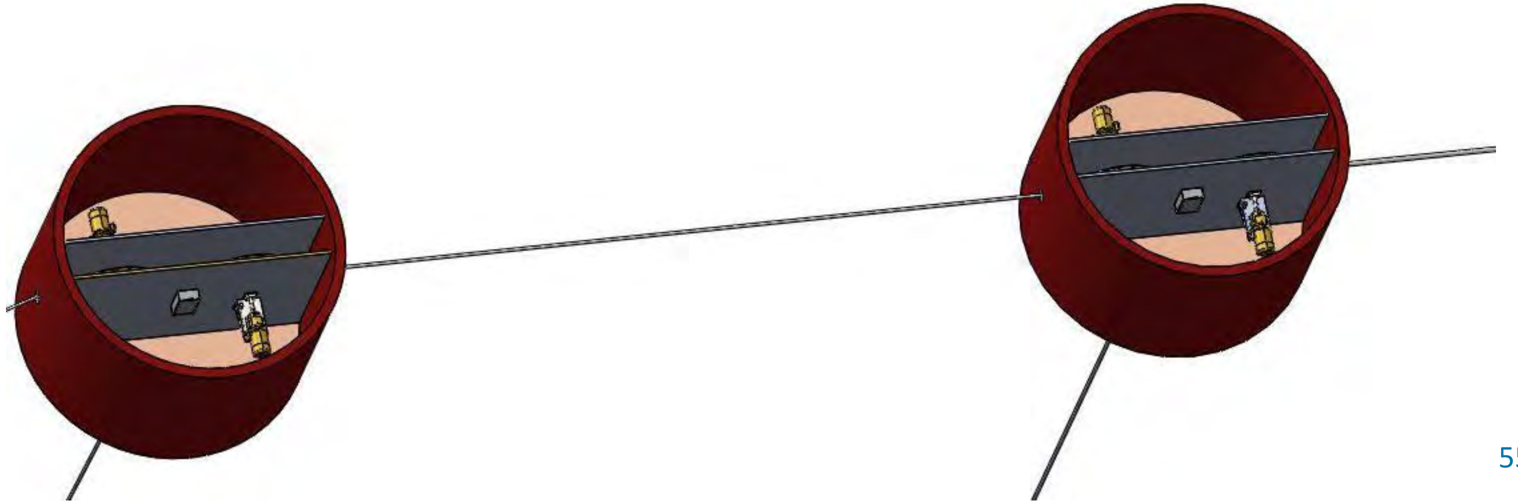
Combined with offshore wind



Interconnected array

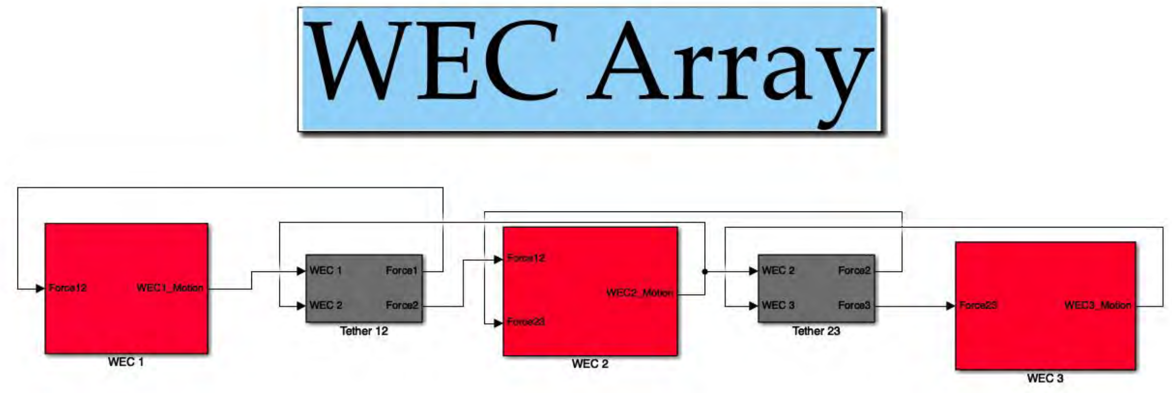


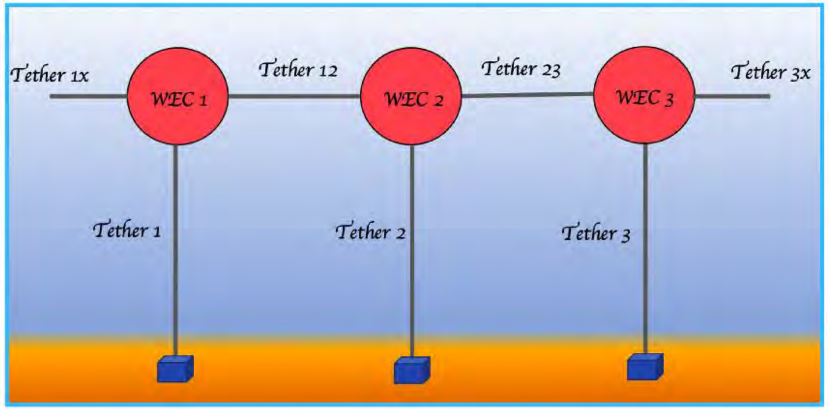




Interconnected array

Description





1) Regular wave analysis

2) Wave periods: 3 – 12 seconds

3) Wave height: 1 m

4) Spherical geometry

5) Interconnection between the WECs

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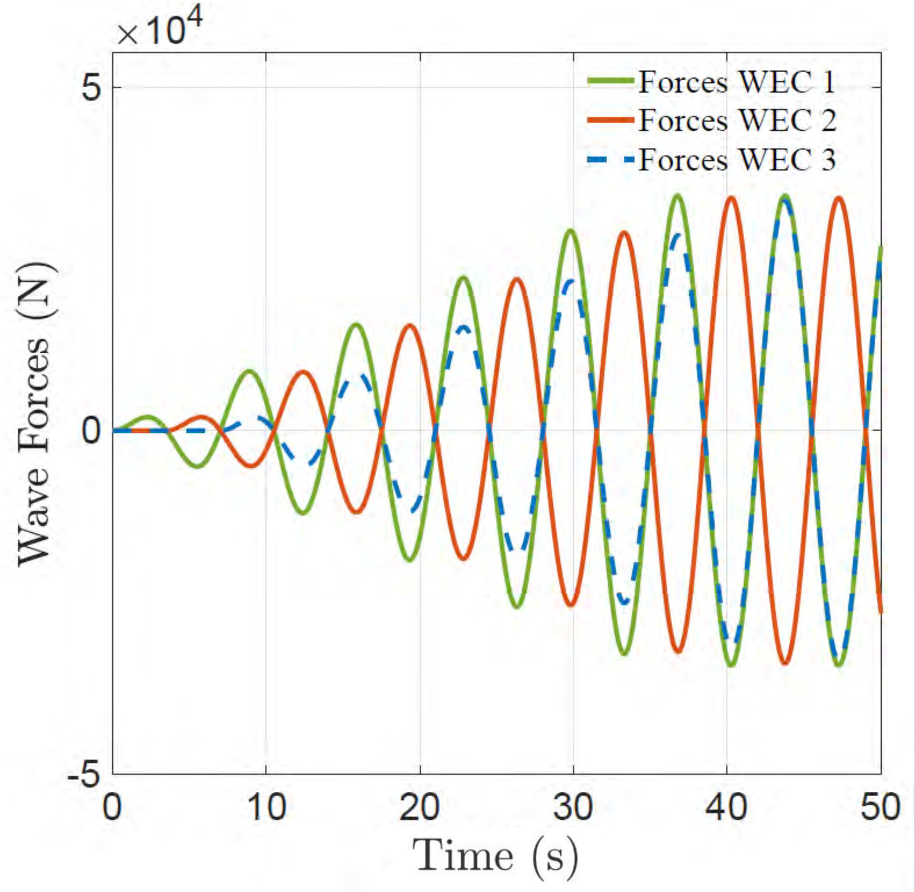
Interconnected array

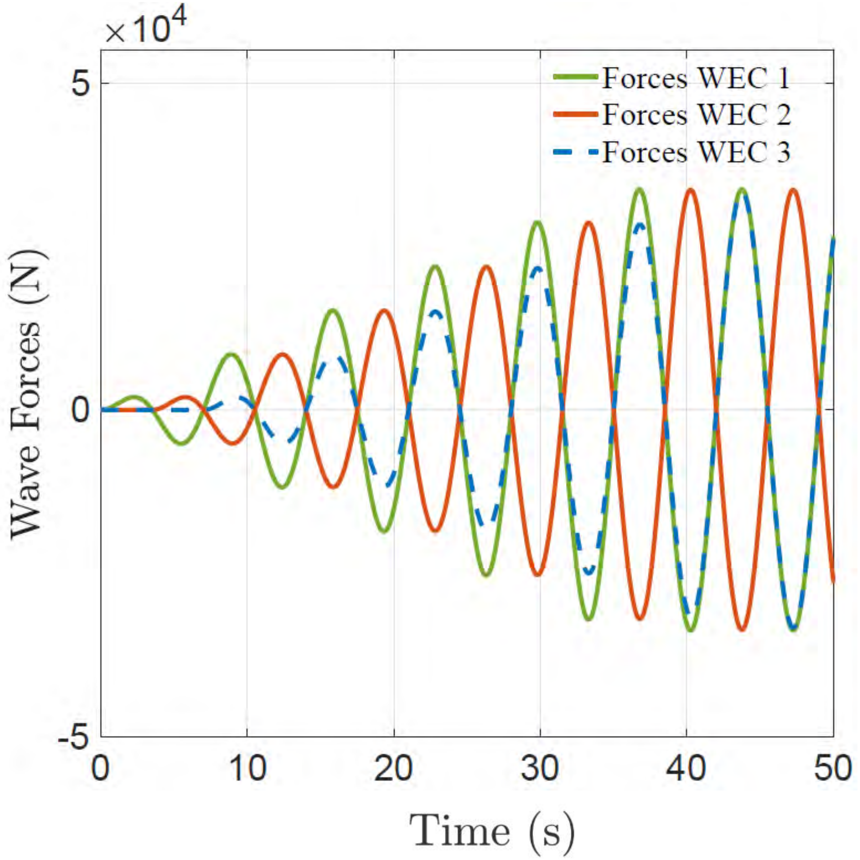
Results

WEC array – T=7 s, H=1 m

Surge forces

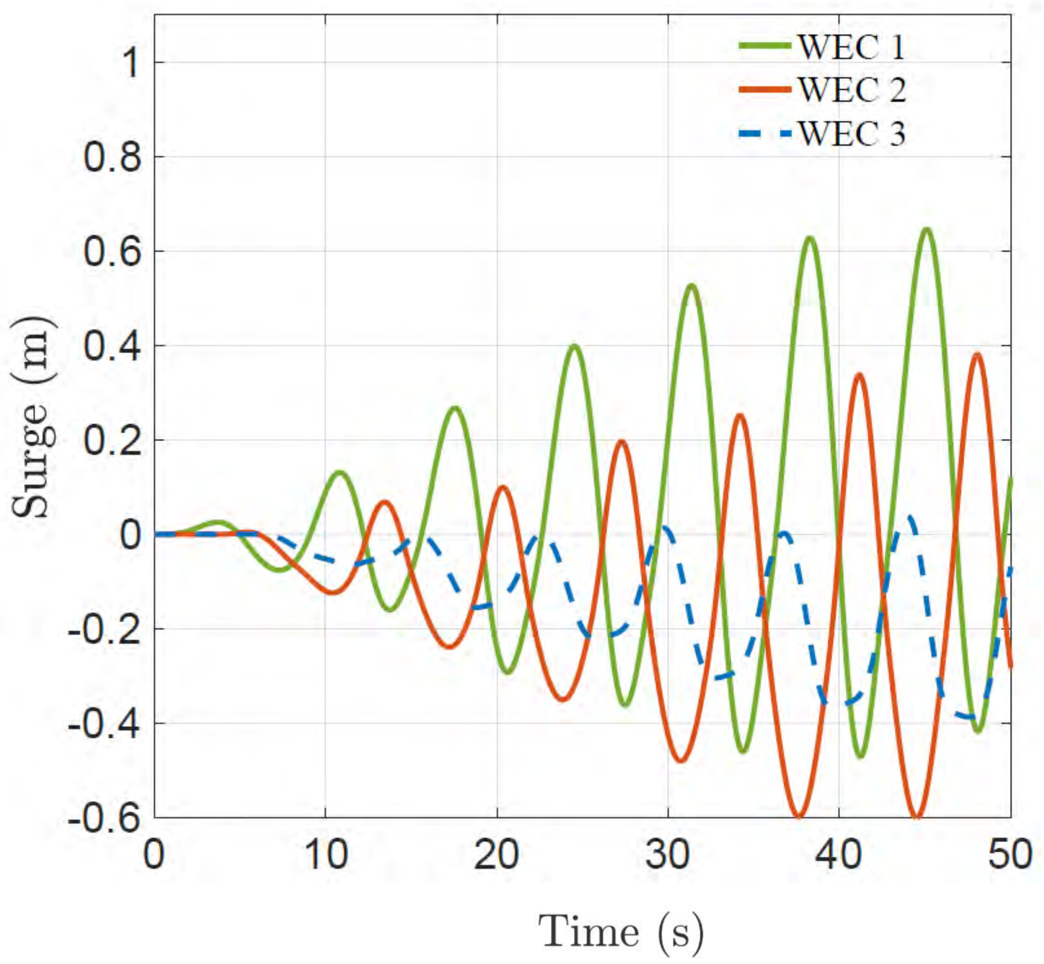


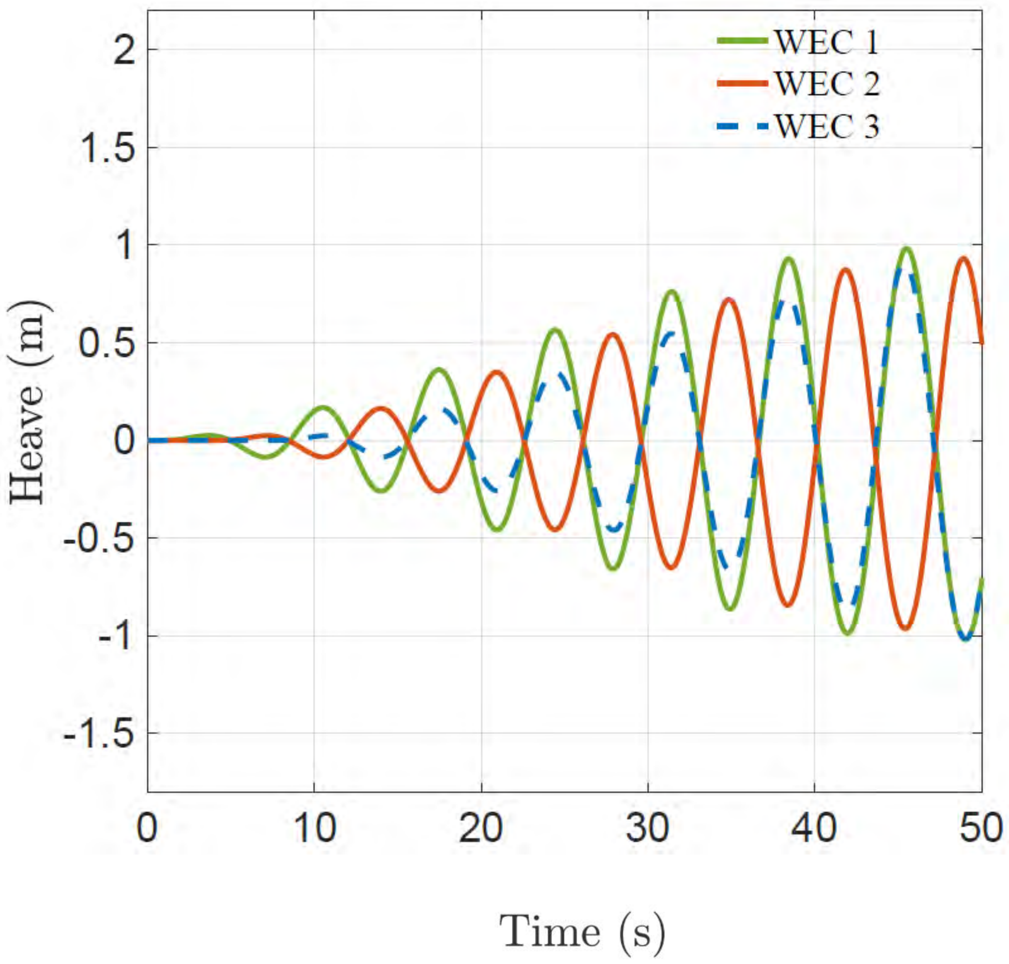
Heave forces



Interconnected array

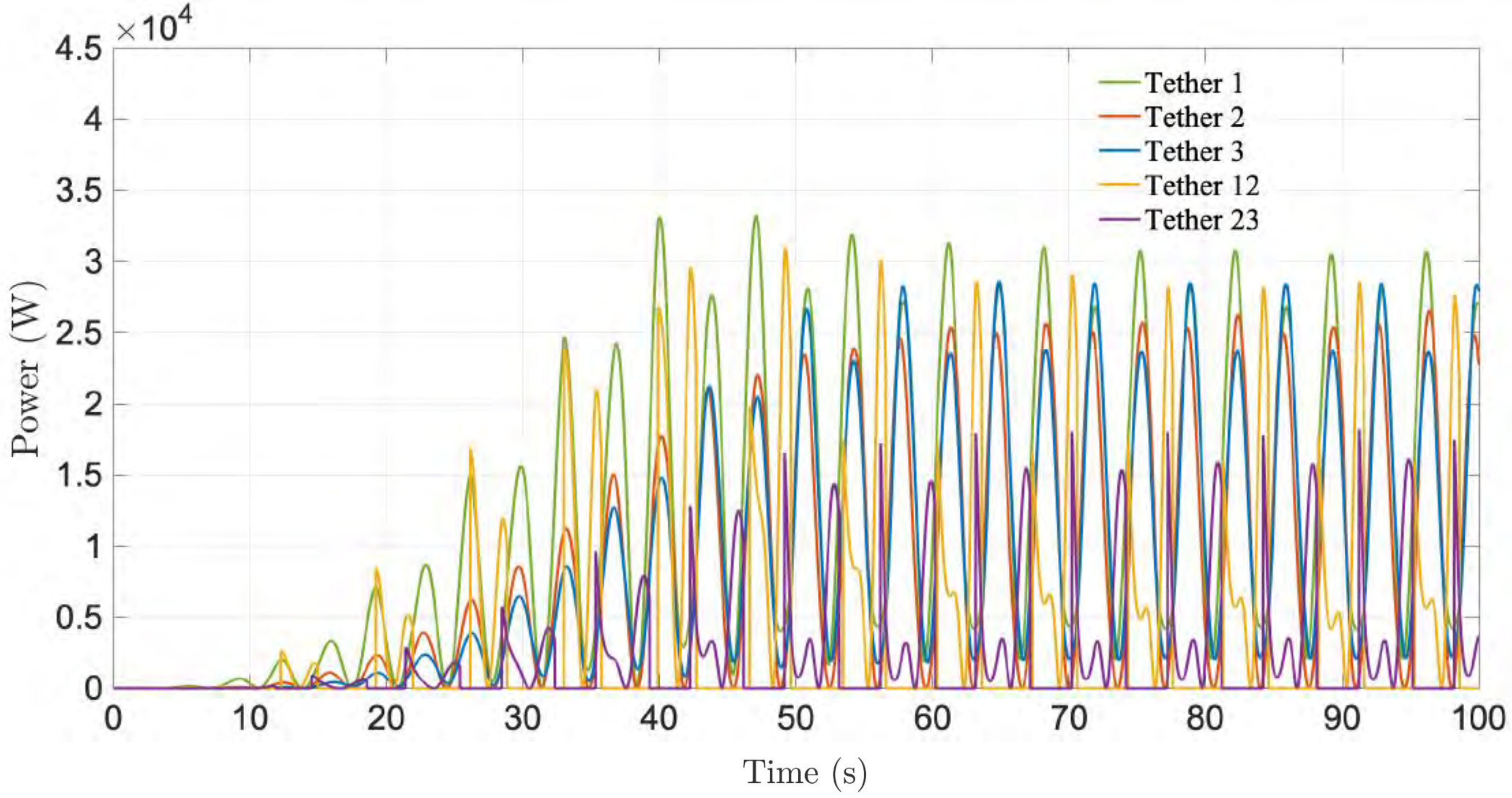
WEC array – T=7 s, H=1 m





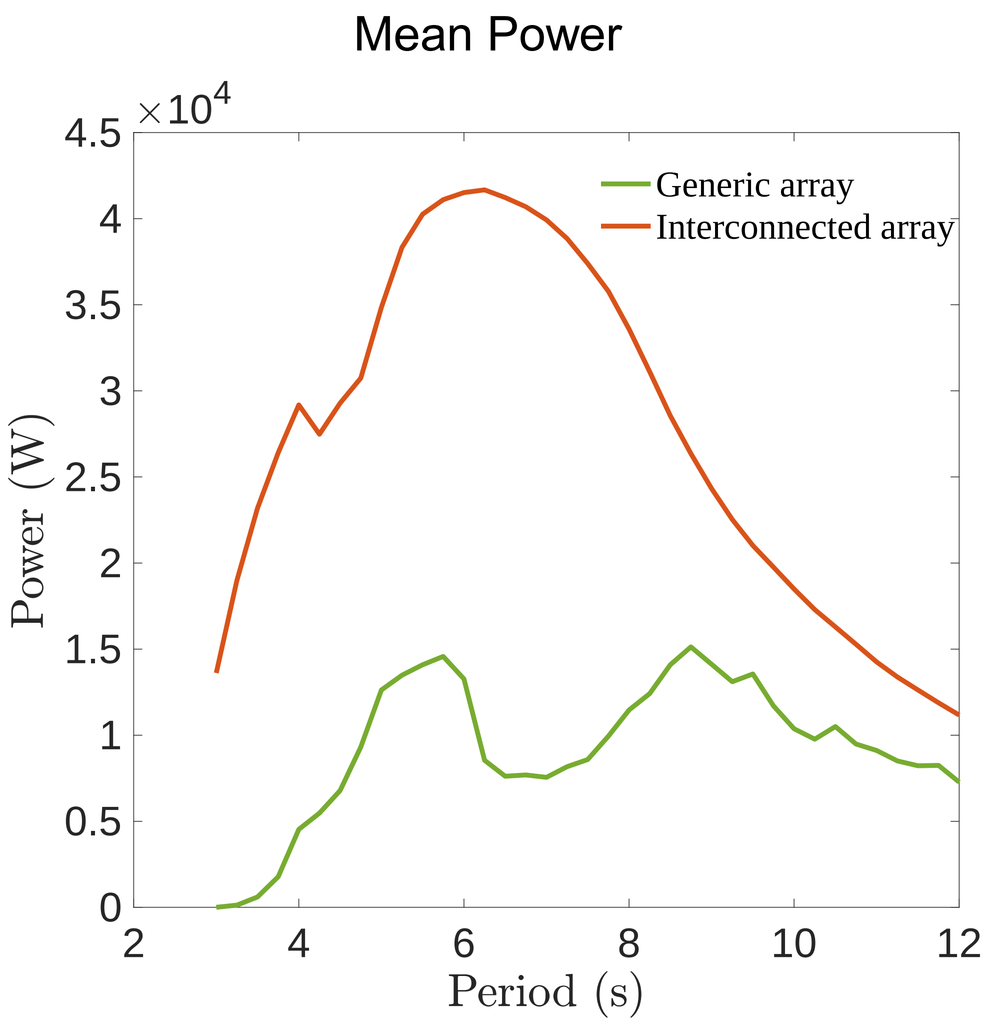
Interconnected array

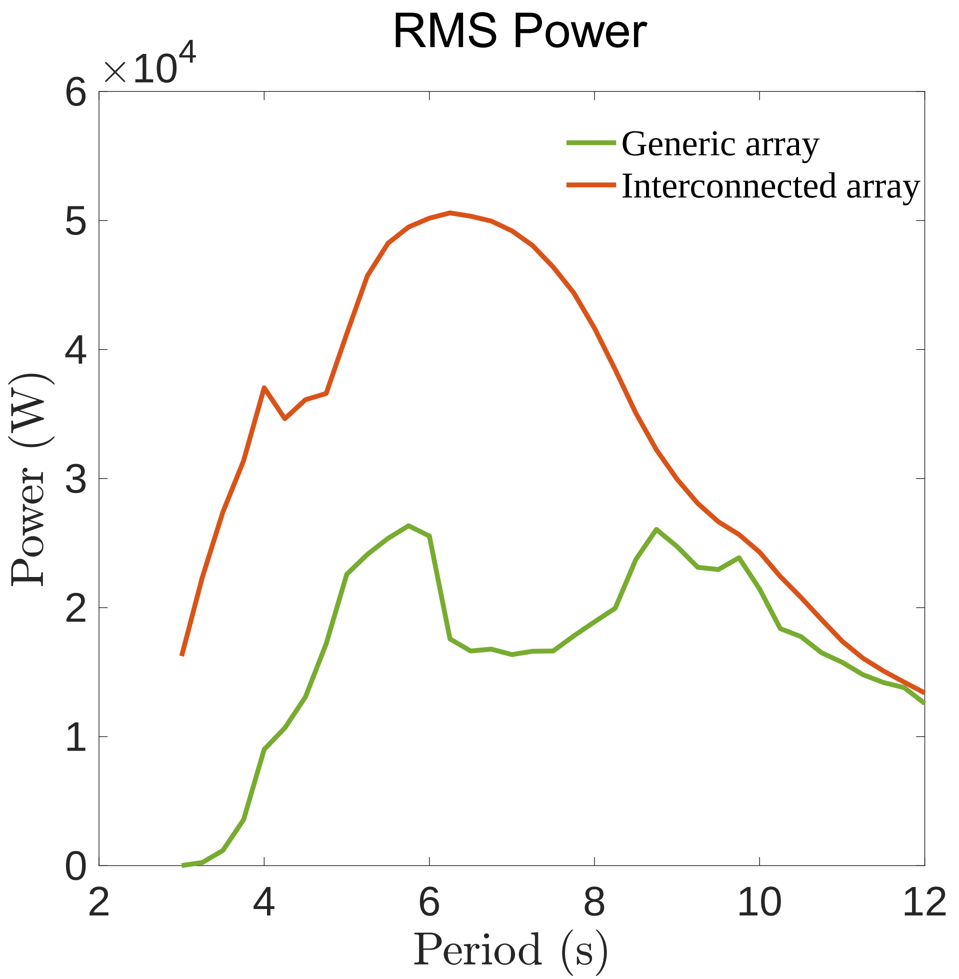
Results



Interconnected array

Results





Conclusions – Thesis results

- Comparison and validation of the model with Ansys AQWA
- Point absorber should not exceed the power capacity of 75 Kw in the Mediterranean Sea (Pantelleria)
- Linear Potential flow theory models overpredict the dynamics
- Linear Potential flow theory results suboptimal PTO coefficients
- Converters with low mass density have an increased permanent load in their PTO and mooring lines. Moreover, mass density influences the range of resonance periods of the device.
- For higher wave heights, the wave absorption efficiency of the converter decreases

Conclusions – Thesis results

- The numerical results show that except for the self-driving ES algorithm ther other four strategies reliably converge for the two-parameter optimization problem
- All extremum seeking schemes achieve optimum within a single simulation
- A point absorber, which is able to control its vertical position under the sea is able to avoid extreme wave conditions and continue to function under the desired wave energy flux
- The multi tether PS teen to behave and perfom exactly as the generic PA when the length of the main mooring is greater than 10 m
- The virtual seabed can contribute significantly to the power performance of the device since the lateral moorings - PTOs can absorb wave energy from the surge motion

Conclusions – Thesis results

- The interconnection between the point absorbers in a WEC array can result higher power performance in comparison to a generic point absorber array

Thanks for your attention

Q&A

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