Nonlinear Analysis of Structures using Unified Formulations

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Overview

In this research, Advanced Finite Element methods are used to describe the mechanical nonlinear behavior of complex structures and materials such as anatomic components. Both geometrical and physical nonlinearities will be considered using the Component-Wise approach (CW) in order to obtain the accurate and precise results by CUF-1D or CUF-2D nonlinear models.
The first ongoing part of this study focuses on the geometrical nonlinearities such as the large deflection and post-buckling behavior of structures specifically isotropic rectangular plates. By taking into account the three-dimensional Green–Lagrange strain components, the explicit forms of the secant and tangent stiffness matrices of unified plate elements are presented in terms of the fundamental nuclei and nonlinear parameters. The Newton–Raphson linearization scheme combined with a path-following method based on the arc-length constraint is utilized to solve the geometrically nonlinear problem. Nonlinear CUF-2D plate model is used considering different nonlinear theories based on Green–Lagrange strain components. In this regard, the well-known von Kármán theory for the nonlinear deformations of plates is focused with different modification such as the thickness stretching and shear deformations due to transverse deflection. The post-buckling curves and related stress distributions for each case are presented and discussed.
The second part of this PhD thesis will be focused on the implementation of Physical Nonlinearities in the CUF-1D or CUF-2D models in order to be used in the complex biostructures or soft materials with different plastic or hyperelastic behaviors.

FN in Unified Formulations

If we use the indicial notation for the theory of structures approximation and the calculation of Stiffness matrix based on the Finite Element method, we have the following equations in matrix form

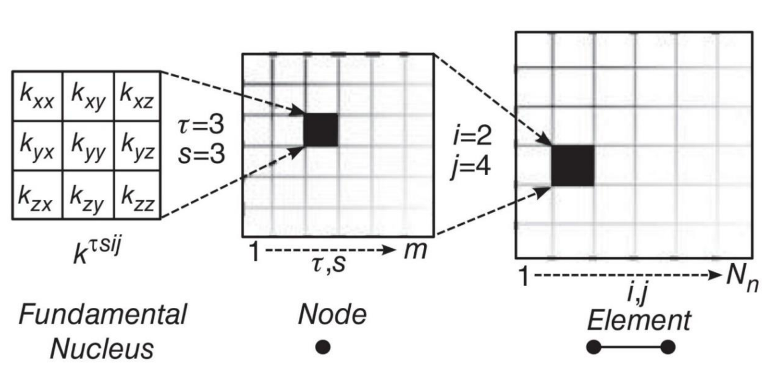
$$\{\varepsilon\} = [B_i]\{S\} \quad , \quad \{S\} = \begin{bmatrix} S^i_{\tau x} \\ S^i_{\tau y} \\ S^i_{\tau z} \end{bmatrix} \quad , \quad [B_i] = \begin{bmatrix} N_i^\tau F^i_{\tau x,x} & 0 & 0 \\ 0 & N^\tau_{i,y} F^i_{\tau y} & 0 \\ 0 & 0 & N_i^\tau F^i_{\tau z,z} \\ N_i^\tau F^i_{\tau x,z} & 0 & N_i^\tau F^i_{\tau z,x} \\ 0 & N_i^\tau F^i_{\tau y,z} & N^\tau_{i,y} F^i_{\tau z} \\ N^\tau_{i,y} F^i_{\tau x} & N_i^\tau F^i_{\tau y,x} & 0 \end{bmatrix}$$

If we introduce the indexes j and s for virtual variations and considerthe $[C]$ matrix for the isotropic material, based on the PVD (Priciple of Virtual Displacement), we have

$$K_{\tau sij} = \int_V [B_j]^T \, [C] \, [B_i] dV \quad , \quad K_{\tau sij} = \begin{bmatrix} k^{\tau sij}_{xx} & k^{\tau sij}_{xy} & k^{\tau sij}_{xz} \\ k^{\tau sij}_{yx} & k^{\tau sij}_{yy} & k^{\tau sij}_{yz} \\ k^{\tau sij}_{zx} & k^{\tau sij}_{zy} & k^{\tau sij}_{zz} \end{bmatrix}$$

$$k^{\tau sij}_{xx} = C_{11} \int_A F^j_{sx,x} F^i_{\tau x,x} dA \int_y N_i^\tau N_j^s dy + C_{44} \int_A F^j_{sx,z} F^i_{\tau x,z} dA \int_y N_i^\tau N_j^s dy + C_{44} \int_A F^j_{sx} F^i_{\tau x} dA \int_y N^\tau_{i,y} N^s_{j,y} dy$$
$$k^{\tau sij}_{xy} = C_{12} \int_A F^j_{sx,x} F^i_{\tau y} dA \int_y N^\tau_{i,y} N_j^s dy + C_{44} \int_A F^j_{sx} F^i_{\tau y,x} dA \int_y N_i^\tau N^s_{j,y} dy$$

that is a 3*3 matrix of Fundamental Nucleus (FN) based on the unified formulations for a given i, j pair.



Unified Formulation of geometrical Nonlinerity

The Green-Lagrange strain vector in terms of the displacement components can be written as

$$\boldsymbol{\epsilon} = \boldsymbol{\epsilon}_l + \boldsymbol{\epsilon}_{nl} = (\mathbf{b}_l + \mathbf{b}_{nl})\mathbf{u},$$

where the linear and nonlinear differential operators $\mathbf{b}_l$ and $\mathbf{b}_{nl}$ are defined as:

$$\mathbf{b}_l = \begin{bmatrix} \partial_x & 0 & 0 \\ 0 & \partial_y & 0 \\ 0 & 0 & \partial_z \\ \partial_z & 0 & \partial_x \\ 0 & \partial_z & \partial_y \\ \partial_y & \partial_x & 0 \end{bmatrix}, \qquad \mathbf{b}_{nl} = \begin{bmatrix} P_{11}\frac{1}{2}(\partial_x)^2 & P_{12}\frac{1}{2}(\partial_x)^2 & P_{13}\frac{1}{2}(\partial_x)^2 \\ P_{21}\frac{1}{2}(\partial_y)^2 & P_{22}\frac{1}{2}(\partial_y)^2 & P_{23}\frac{1}{2}(\partial_y)^2 \\ P_{31}\frac{1}{2}(\partial_z)^2 & P_{32}\frac{1}{2}(\partial_z)^2 & P_{33}\frac{1}{2}(\partial_z)^2 \\ P_{41}\partial_x\partial_z & P_{42}\partial_x\partial_z & P_{43}\partial_x\partial_z \\ P_{51}\partial_y\partial_z & P_{52}\partial_y\partial_z & P_{53}\partial_y\partial_z \\ P_{61}\partial_x\partial_y & P_{62}\partial_x\partial_y & P_{63}\partial_x\partial_y \end{bmatrix},$$

in which $\partial_x = \partial(\cdot)/\partial x$, $\partial_y = \partial(\cdot)/\partial y$, $\partial_z = \partial(\cdot)/\partial z$, and P_{11} to P_{63} are the nonlinear parameters used as coefficients of nonlinear differential operator matrix.

The nonlinear geometric relations for von Kármán strains are as follows.

$$\varepsilon_{xx_{nl}} = \tfrac{1}{2}(u_{z,x})^2 \quad , \quad \varepsilon_{yy_{nl}} = \tfrac{1}{2}(u_{z,y})^2 \quad , \quad \varepsilon_{xy_{nl}} = u_{z,x}u_{z,y}$$

Therefore, for the case of von Kármán nonlinear plate, all the non-linear parameters are zero except P_{11}, P_{12}, and P_{16}. The effect of nonlinear terms on the large-deflection and post-buckling of plates are focused based on different nonlinear models. The matrix of Nonlinear parameters based on these nonlinear models are illustrated in the following figure.

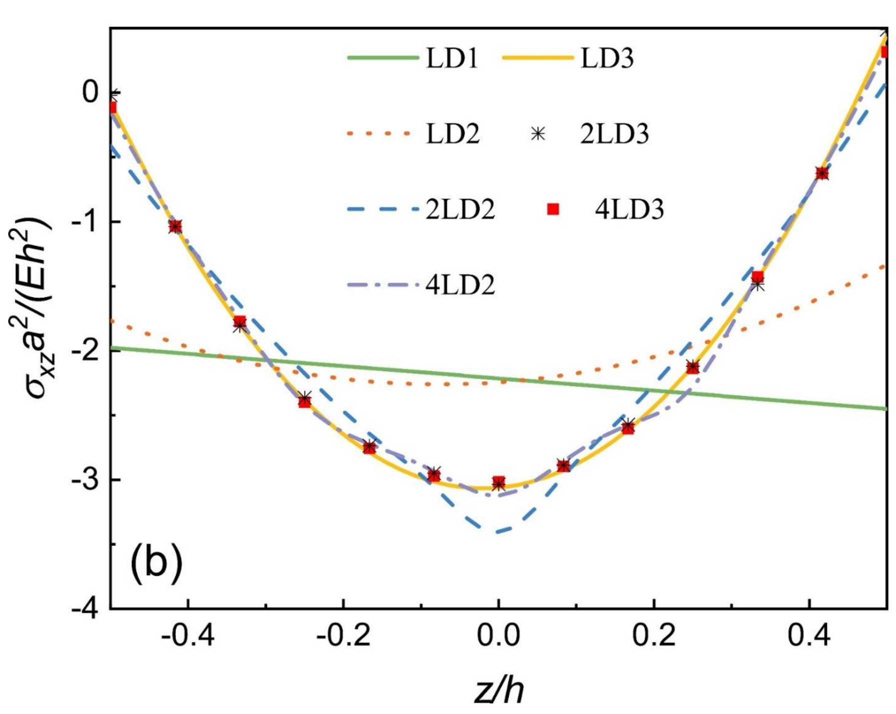
Future Work

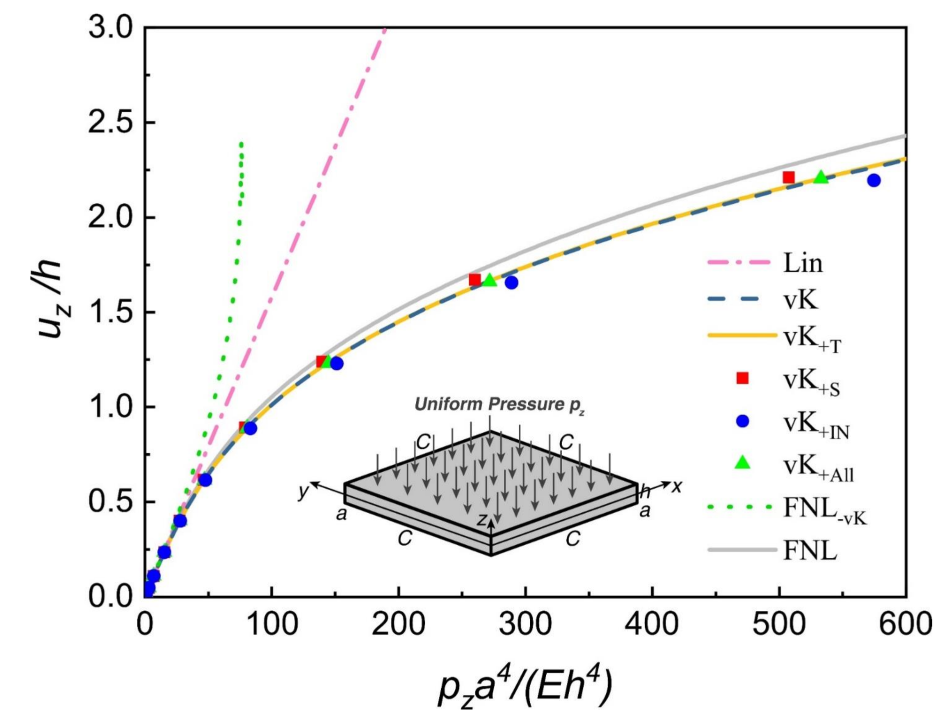
Modeling of physical nonlinear materials requires the use of constitutive equations, which in the simplest form account for nonlinear elasticity. The challenge in mathematical modeling is to select or develop an appropriate formulation and to experimentally determine associated model parameters.

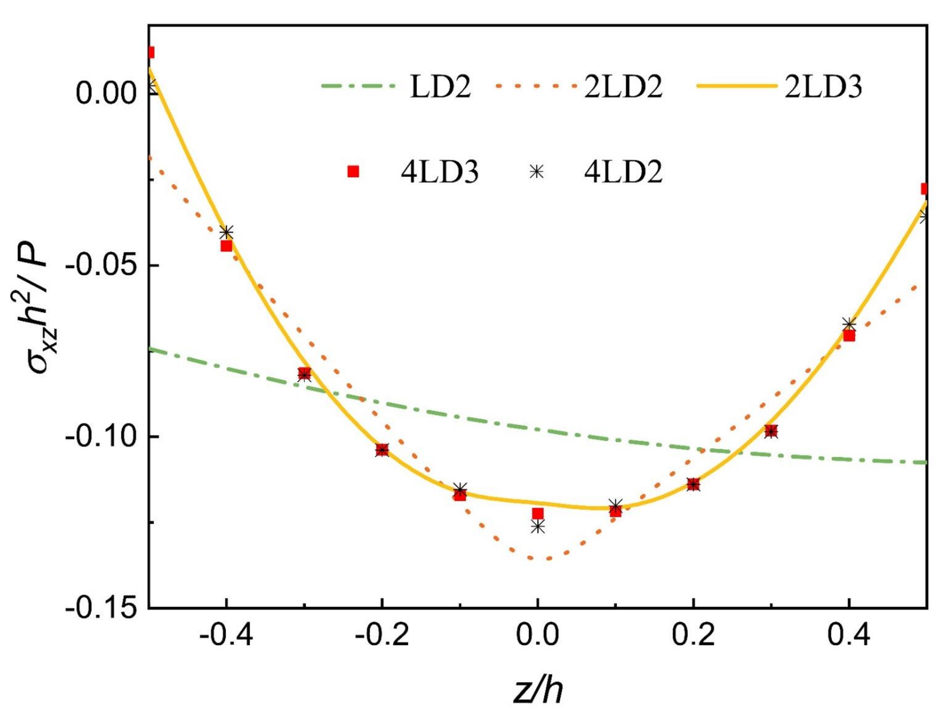
Highly deformable elastic materials are found abundantly in living systems such as arterial wall, skin, tendon, muscle and cornea. Therefore, in the next steps pf this thesis, equations governing a continuum, resulting from the conservation of mass and energy and balance of momenta, will be derived, and constitutive relations will be investigated. Then a mathematical framework describing the governing constitutive equations will be developed and implemented it in the CUF 1D or CUF 2D nonlinear models.

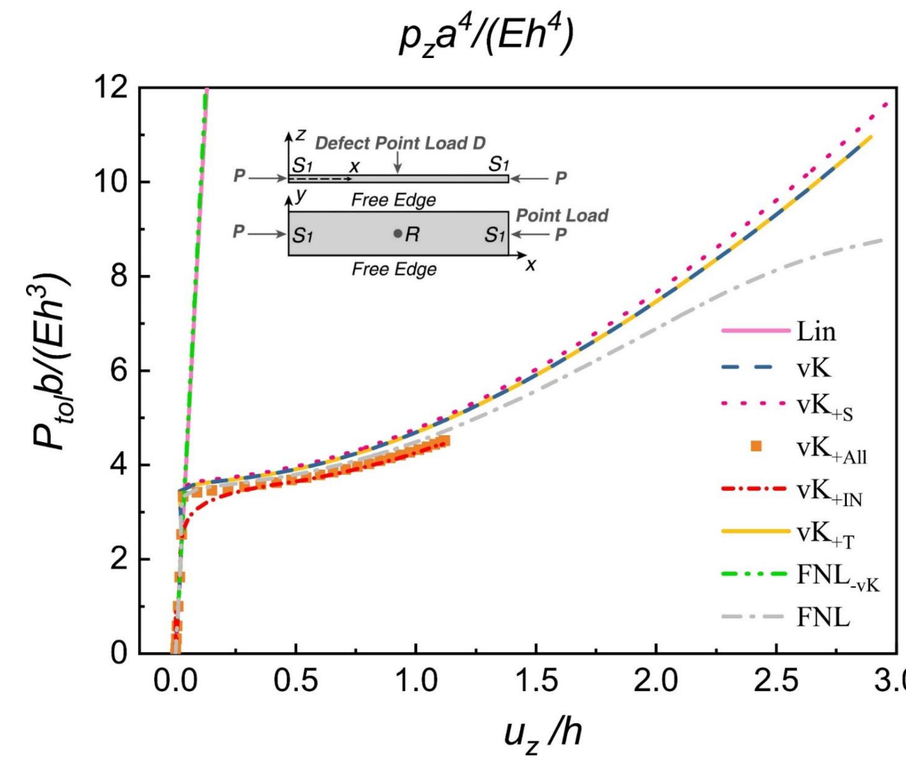
Evaluation of geometrically nonlinear parameters

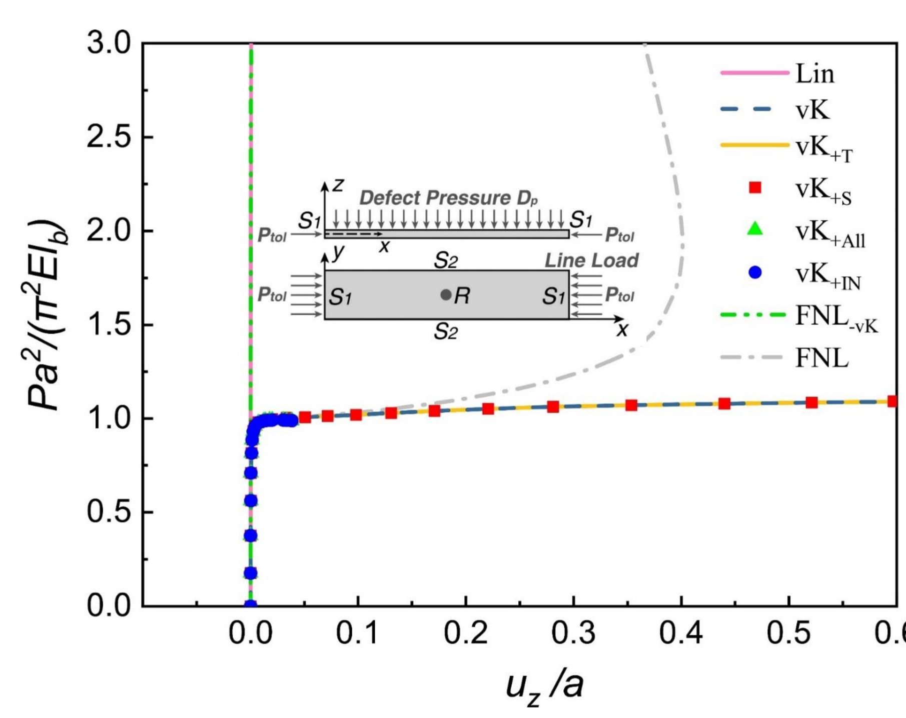
Theory	Description	Notation	Theory	Description	Notation
$\begin{pmatrix} \circ & \circ & \circ \\ \circ & \circ & \circ \\ \circ & \circ & \circ \\ \circ & \circ & \circ \\ \circ & \circ & \circ \\ \circ & \circ & \circ \end{pmatrix}$	*Linear*	Lin	$\begin{pmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \circ & \circ & \circ \\ \circ & \circ & \circ \\ \circ & \circ & \circ \\ \bullet & \bullet & \bullet \end{pmatrix}$	*von Kármán considering in-plane components of displacement*	vK_{+IN}
$\begin{pmatrix} \circ & \circ & \bullet \\ \circ & \circ & \bullet \\ \circ & \circ & \circ \\ \circ & \circ & \circ \\ \circ & \circ & \circ \\ \circ & \circ & \bullet \end{pmatrix}$	*von Kármán*	vK	$\begin{pmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \circ & \circ & \bullet \\ \circ & \circ & \bullet \\ \circ & \circ & \bullet \\ \bullet & \bullet & \bullet \end{pmatrix}$	*von Kármán with all the mentioned considerations*	vK_{+All}
$\begin{pmatrix} \circ & \circ & \bullet \\ \circ & \circ & \bullet \\ \circ & \circ & \bullet \\ \circ & \circ & \circ \\ \circ & \circ & \circ \\ \circ & \circ & \bullet \end{pmatrix}$	*von Kármán considering thickness stretching*	vK_{+T}	$\begin{pmatrix} \bullet & \bullet & \circ \\ \bullet & \bullet & \circ \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \circ \end{pmatrix}$	*Full Nonlinear without von Kármán terms*	FNL_{-vK}
$\begin{pmatrix} \circ & \circ & \bullet \\ \circ & \circ & \bullet \\ \circ & \circ & \circ \\ \circ & \circ & \bullet \\ \circ & \circ & \bullet \\ \circ & \circ & \bullet \end{pmatrix}$	*von Kármán considering shear deformations due to transverse deflection*	vK_{+S}	$\begin{pmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{pmatrix}$	*Full Nonlinear*	FNL











Courses:

- Modelli agli elementi finiti avanzati per problemi meccanici e multicampo
- Writing Scientific Papers in English
- Project Management
- Public speaking
- Communication
- Strumenti e tecnologie per lo sviluppo del prodotto
- Aspetti avanzati del metodo degli elementi finite
- Aeroelastic tailoring - modelling, design, manufacturability and experiments (didattica di eccellenza)
- Materials by design - How structure meets function (didattica di eccellenza)
- MUL2 Spring School on Virtual Manufacturing and Testing of Composites