

Refined One Dimensional Finite Element models applied to bio-mechanical problems



Daniele Guarnera
PhD Student – XXXI Ciclo – DIMEAS
Supervisors: Prof. E. Carrera, Dr. A. Pagani



The **Mul2** is a research team in which advanced models are developed and employed in different analyses and applications. The research group is devoted to the implementation of reduced models by using of the *Carrera Unified Formulation* (CUF):

Structural Analyses

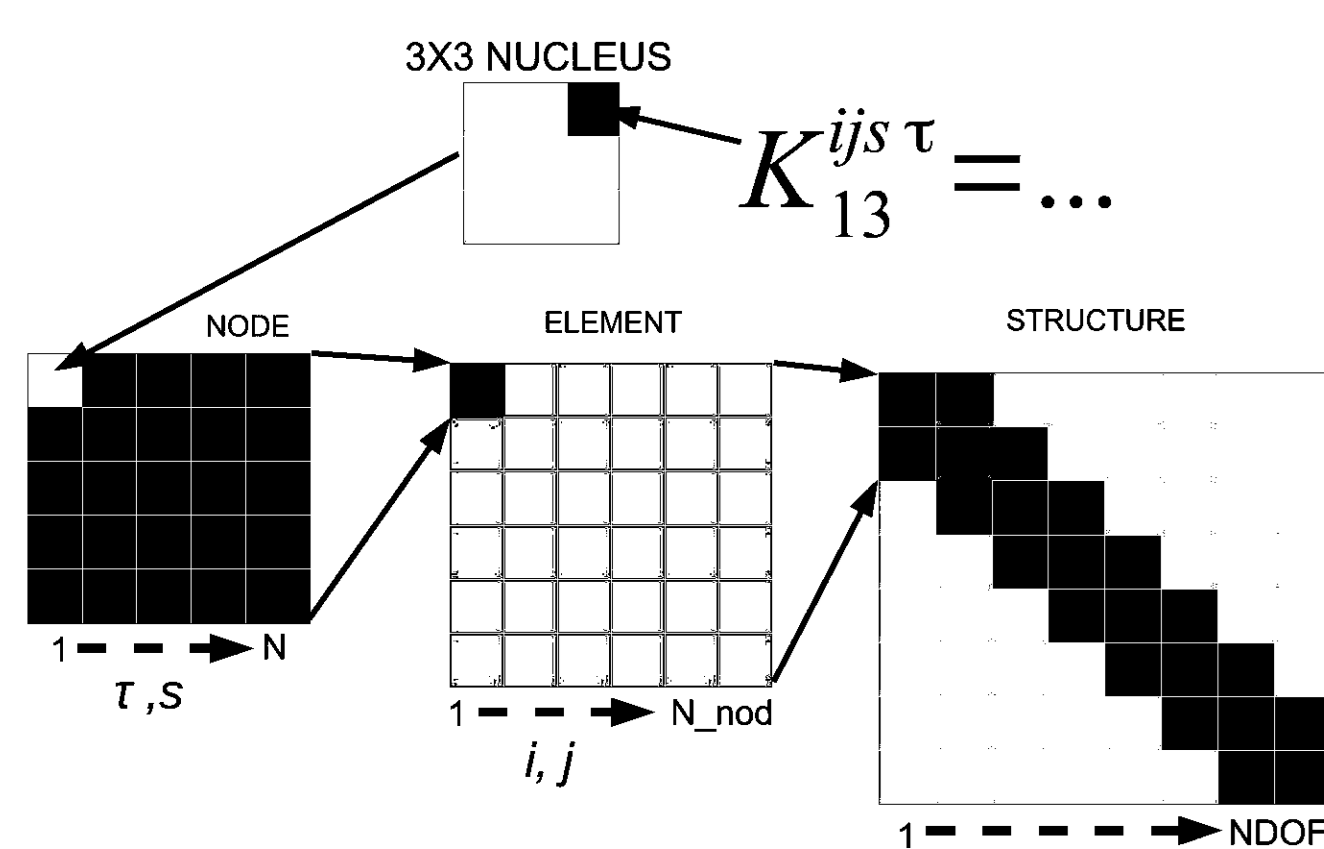
CUF-1D of the beam cross-section displacement field is described by an expansion of generic function F_τ and than variables are interpolated along the axis (y) by using the *shape functions* N_i . The problem may be governed by the *Principle of Virtual Displacement*:

$$\delta L_{int} = \delta \mathbf{q}_{\tau i}^T \mathbf{K}^{ijrs} \mathbf{q}_{sj}$$

$$\delta L_{ine} = \delta \mathbf{q}_{\tau i}^T \mathbf{M}^{ijrs} \mathbf{q}_{sj}$$

$$\delta L_{ext} = \delta \mathbf{q}_{\tau i}^T \mathbf{P}^{ir}$$

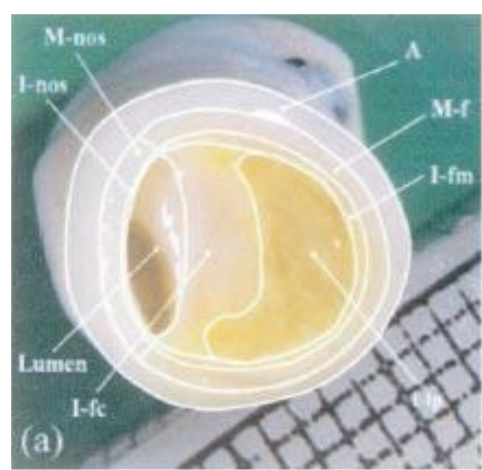
Where \mathbf{K} , \mathbf{M} and \mathbf{P} are written in terms of fundamental nuclei, the core of the technique.



Results

Static and dynamic analysis of an **atherosclerotic plaque** of human artery and of a **dental prosthesis**

Human artery affected by atherosclerotic plaque under effect of blood pressure.

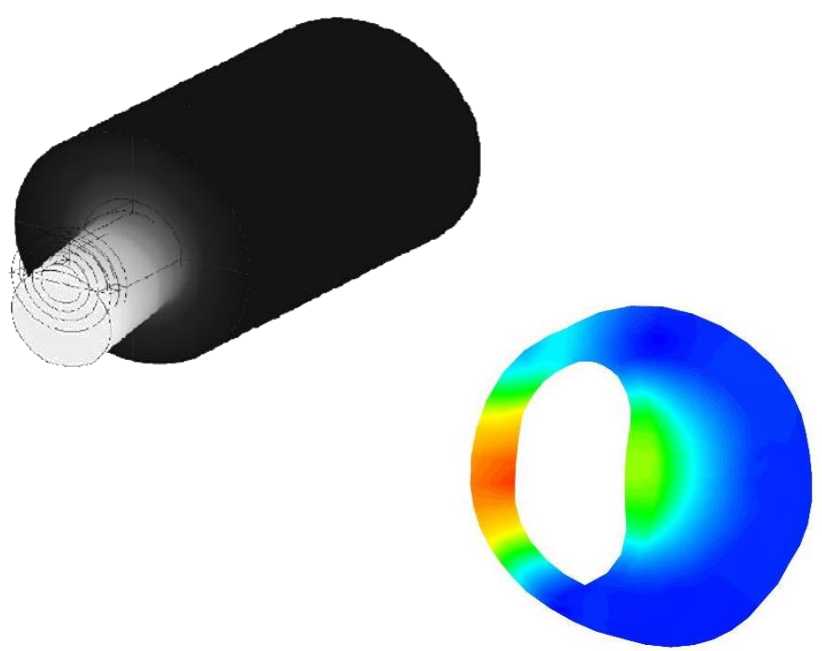


Dental implant inside human gingiva and bone under effect of masticatory force

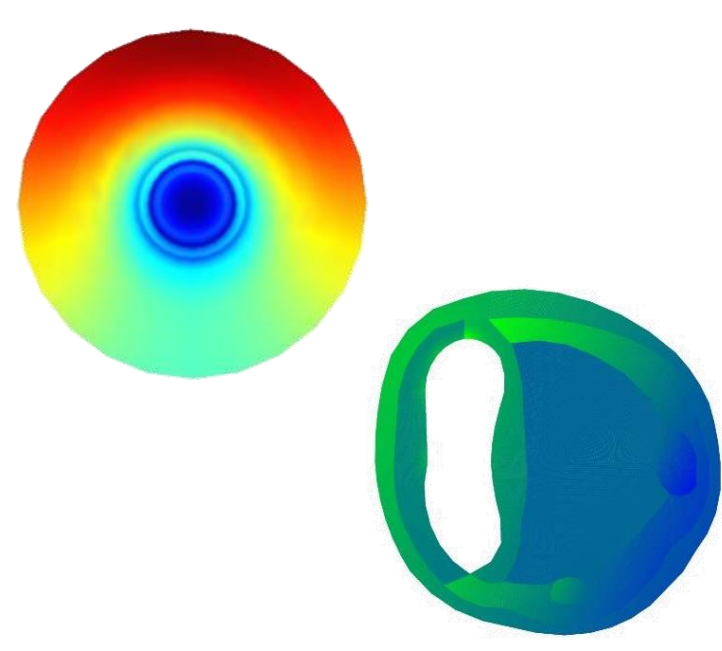


In the framework of the Unified Formulation, it is possible enhance the accuracy of results by increasing the order of the expansion function across the section. Due to this capability, is possible to obtain an accurate solution of displacement, stress and modal shapes with a much lower computational effort.

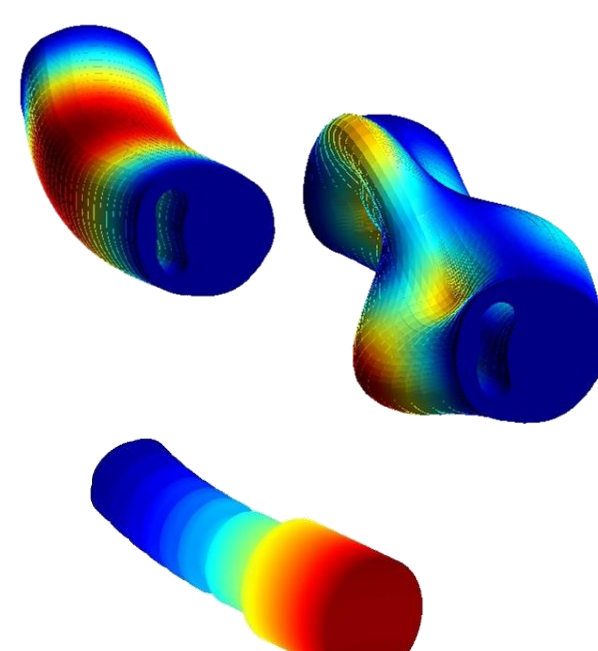
Displacements



Stress

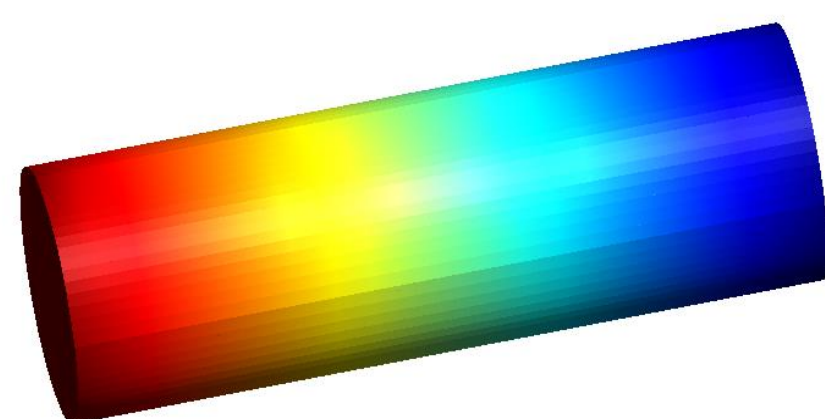


Modal shapes



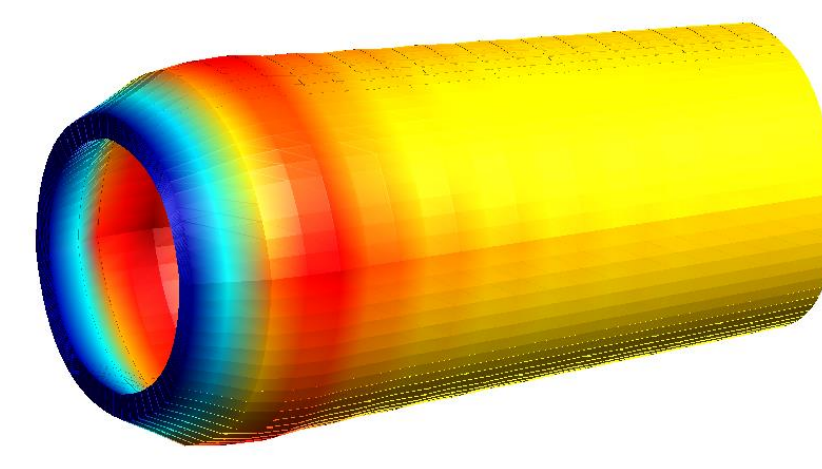
It is demonstrated that the results obtained by one-dimensional CUF models are in agreement with those got with commercial FE software and those present in literature.

Pressure trends obtained via 1D-CUF models for Fluid-Dynamic



Fluid-Structure Interaction

The idea is to exploit the advantages of both **one-dimensional** models to analyse the **fluid-structure interaction** in a unified and smart manner.



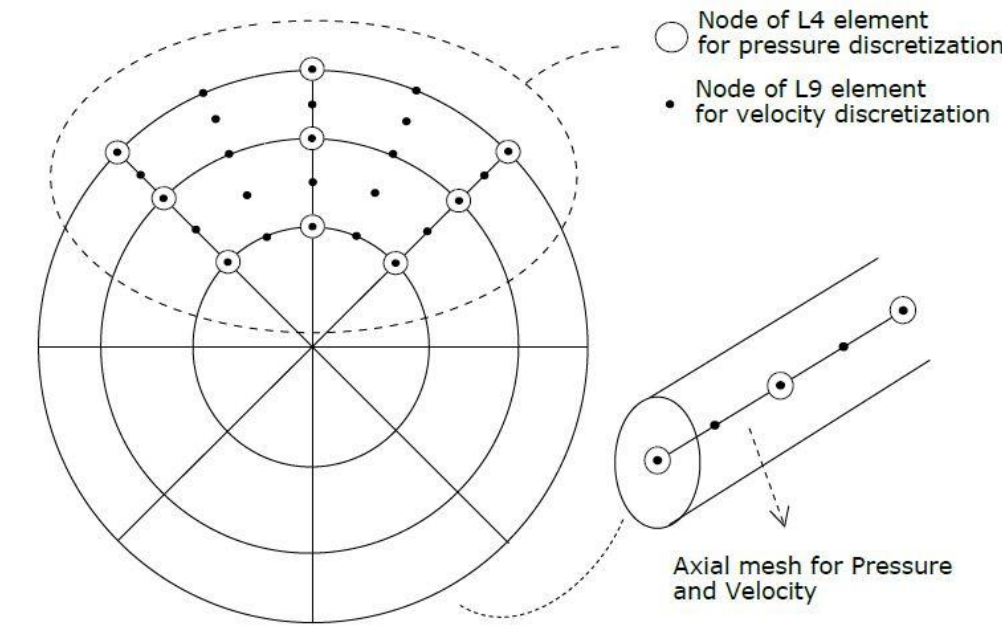
Structural response obtained via 1D-CUF models for Structure

CFD Analyses

According to **CUF**, the velocity field \mathbf{u}_h and the pressure field p_h are expressed as a generic expansion of the generalized unknowns through arbitrary functions of the cross-section domain coordinates while along the pipe axis, the generalized velocities $\mathbf{u}_\tau(y)$ and pressures $p_m(y)$ are described as a function of the unknown nodal vectors, $\mathbf{u}_{\tau i}$ and p_{mt} , and the 1D shape functions, N_i and N_t .

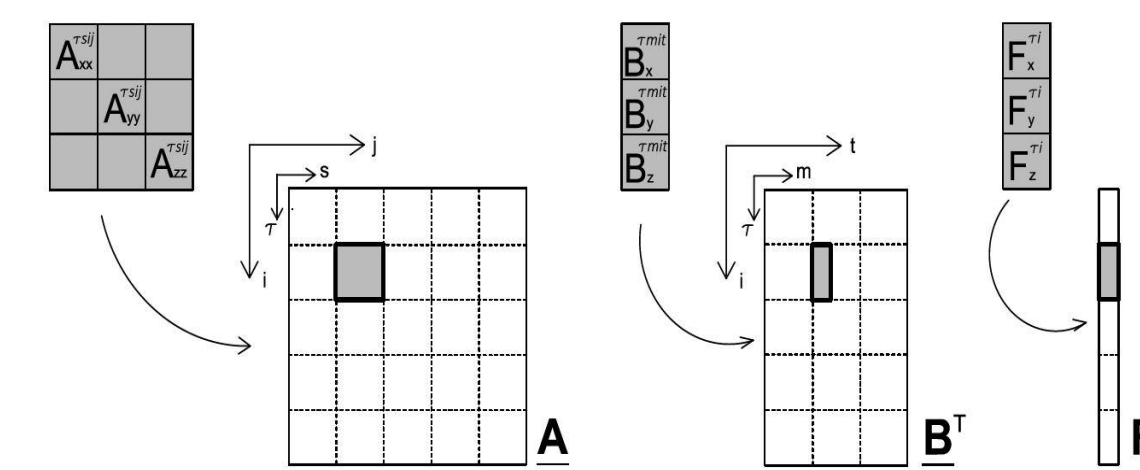
$$\mathbf{u}_h(x, y, z) = F_\tau^U(x, z) N_i^U(y) \mathbf{u}_{\tau i}$$

$$p_h(x, y, z) = F_m^P(x, z) N_t^P(y) p_{mt}$$



In the case of highly viscous fluids, the Navier-Stokes set of equations for incompressible flow can be reduced to the so-called **Stokes equations**. Adopting the CUF technique, it is possible to derive the matrix form of these equation in terms of fundamental nuclei:

$$\begin{cases} \mathbf{A}^{\tau s i j j} \mathbf{q}_{s j} + \mathbf{B}^{\tau m i t T} p_{m t} = \mathbf{F}^{\tau i} & \forall \tau, \forall i \\ \mathbf{B}^{m s t j j} \mathbf{q}_{s j} = 0 & \forall m, \forall t \end{cases}$$



Results

- First assessment: Poiseuille flow in a cylindrical and non-cylindrical pipe

Axial velocity at midspan: comparison among 1D models, 3D solution and analytical solution.

