Advanced and nonlocal theories for the multi-scale/multi-field analysis of structures

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ScuDo Scuola di Dottorato - Doctoral School WHAT YOU ARE, TAKES YOU FAR

XXXIII Cycle

Joint research project with top university founded by Compagnia di San Paolo (City University of Hong Kong)

MUL2

Methodology

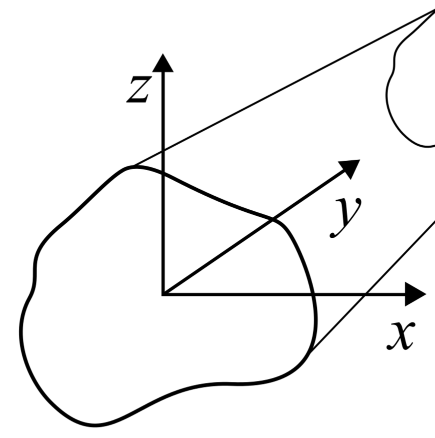
Carrera Unified Formulation allows the three-dimensional displacement field *u(x, y, z)* to be expressed as a general expansion of the primary unknowns. In the case of one-dimensional theories, one has:

$$\boldsymbol{u}(x,y,z) = F_s(x,z)\boldsymbol{u}_s(y), \qquad s = 1,2,\ldots,M$$

F_s: functions of the coordinate *x* and *z* on the cross-section

$\boldsymbol{u}_s$: vector of the generalized displacements, along the beam axis

M: number of the terms used in the expansion.

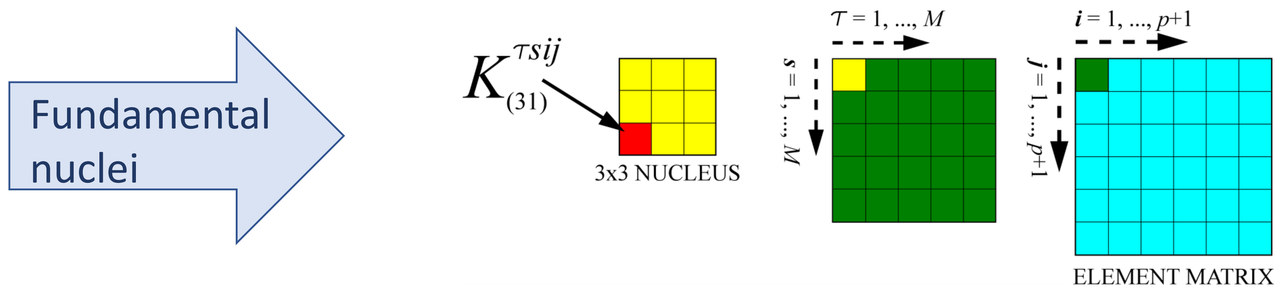


From the principle of virtual work:

$$\delta L_{\mathrm{int}} = \delta \mathbf{q}_{\tau i}^T \mathbf{K}^{ij\tau s}\mathbf{q}_{sj}$$

$$\delta L_{\mathrm{ine}} = \delta \mathbf{q}_{\tau i}\mathbf{M}^{ij\tau s}\ddot{\mathbf{q}}_{sj}$$

$$\delta L_{\mathrm{ext}} = \delta \mathbf{q}_{\tau i}\mathbf{P}^{i\tau}$$



Geometrical Nonlinearity

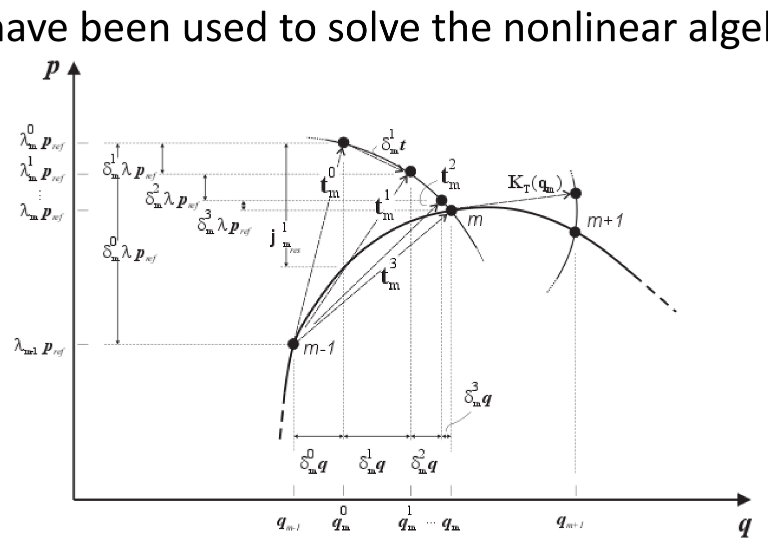
Formulation

$$\boxed{\varepsilon = \varepsilon_l + \varepsilon_{nl} = (\boldsymbol{b}_l + \boldsymbol{b}_{nl})\boldsymbol{u}} \rightarrow \{u_x \;\; u_y \;\; u_z\}^T$$

$$\begin{bmatrix} \partial_x & 0 & 0 \\ 0 & \partial_y & 0 \\ 0 & 0 & \partial_z \\ \partial_z & 0 & \partial_x \\ 0 & \partial_z & \partial_y \\ \partial_y & \partial_x & 0 \end{bmatrix} \qquad \begin{bmatrix} \frac{1}{2}(\partial_x)^2 & \frac{1}{2}(\partial_x)^2 & \frac{1}{2}(\partial_x)^2 \\ \frac{1}{2}(\partial_y)^2 & \frac{1}{2}(\partial_y)^2 & \frac{1}{2}(\partial_y)^2 \\ \frac{1}{2}(\partial_z)^2 & \frac{1}{2}(\partial_z)^2 & \frac{1}{2}(\partial_z)^2 \\ \partial_x\partial_z & \partial_x\partial_z & \partial_x\partial_z \\ \partial_y\partial_z & \partial_y\partial_z & \partial_y\partial_z \\ \partial_x\partial_y & \partial_x\partial_y & \partial_x\partial_y \end{bmatrix}$$

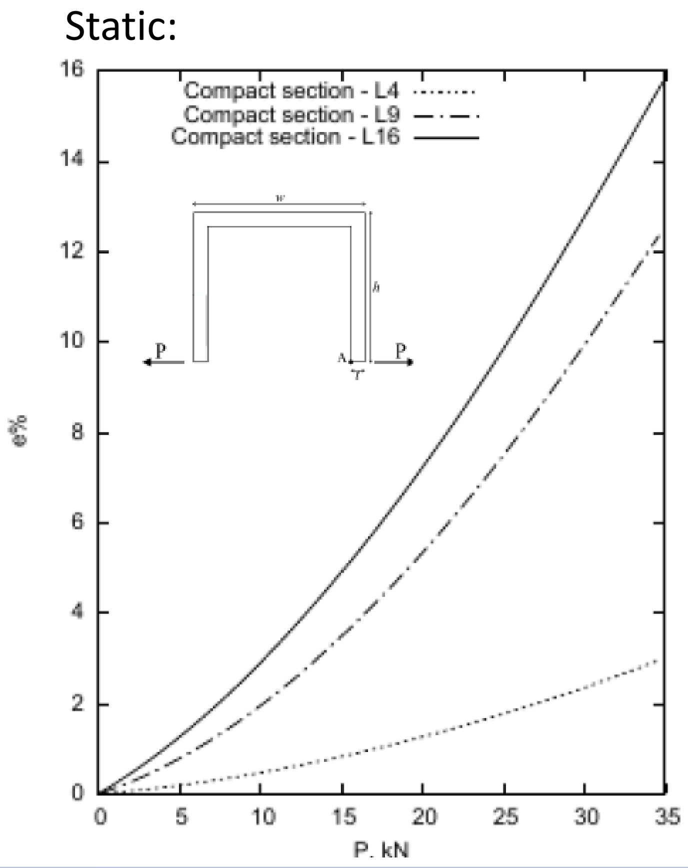
Arc-length

A Newton-Raphson linearized incremental scheme along with an arc-length constraint relation have been used to solve the nonlinear algebraic system.

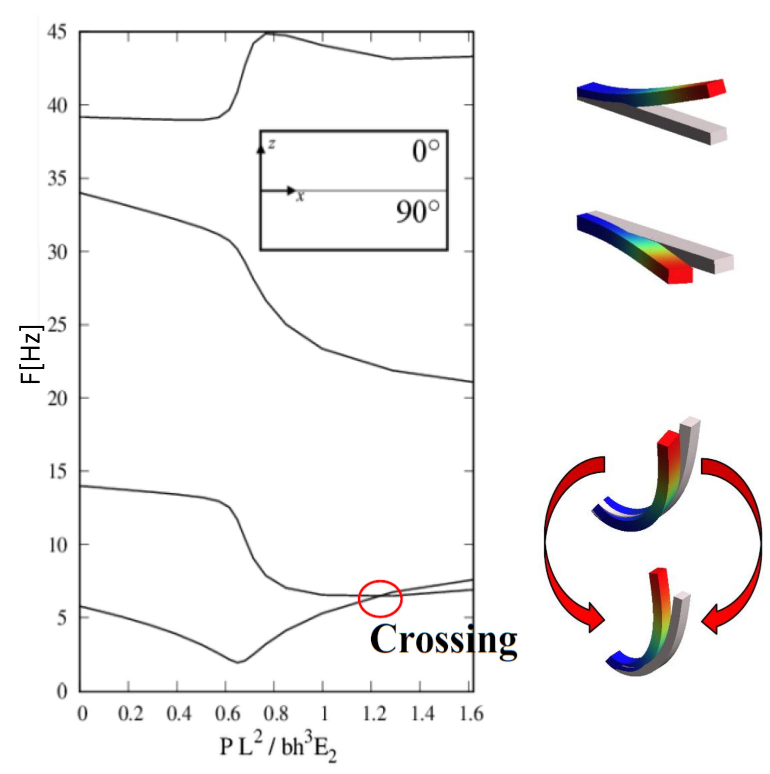


Results

Static:



Dynamic:



Micropolar Elasticity

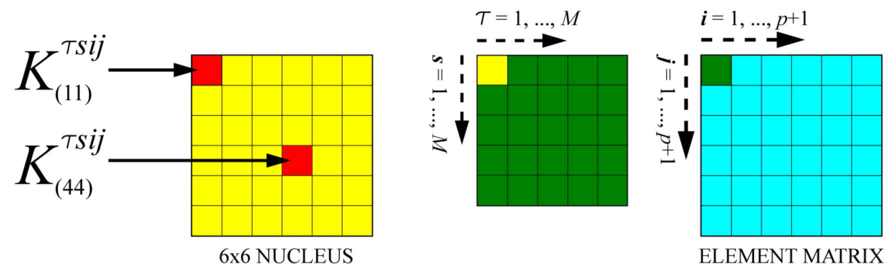
Fundamental nuclei

The displacement field is described by the 3 displacements and 3 microrotations. The unknowns are 6 for each point.

Unknowns:

$$\boldsymbol{u}(1{,}2{,}3) = \{u_1 \;\; u_2 \;\; u_3 \;\; \omega_1 \;\; \omega_2 \;\; \omega_3\}^T$$

Displacements — Micro-Rotations



Micropolar Elasticity

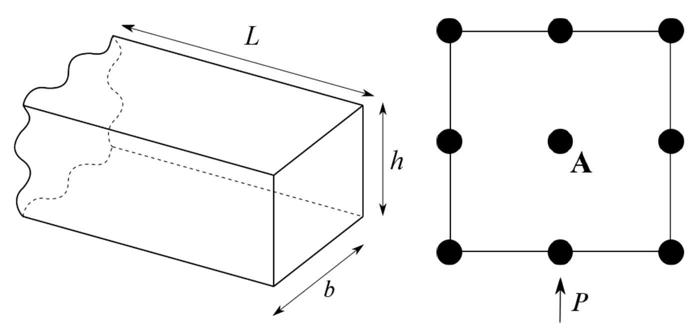
In micropolar elasticity it is assumed that the body consists of interconnected particles in the form of small rigid bodies. The internal forces are defined in terms of a classical force stress tensor $\boldsymbol{\sigma}$ and a micropolar couple stress tensor $\boldsymbol{\mu}$:

$$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{21} & \sigma_{31} \\ \sigma_{12} & \sigma_{22} & \sigma_{32} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{bmatrix} \quad \mu = \begin{bmatrix} \mu_{11} & \mu_{21} & \mu_{31} \\ \mu_{12} & \mu_{22} & \mu_{32} \\ \mu_{13} & \mu_{23} & \mu_{33} \end{bmatrix} \quad \begin{aligned} \sigma &= \lambda(tr\varepsilon)I + (\mu+\alpha)\varepsilon + (\mu-\alpha)\varepsilon^T \\ \mu &= \beta(tr\chi)I + (\gamma+\varepsilon)\chi + (\gamma-\varepsilon)\chi^T \end{aligned}$$

The micropolar deformation is fully described by the asymmetric strain $\boldsymbol{\varepsilon}$ and twist $\boldsymbol{\chi}$ tensors

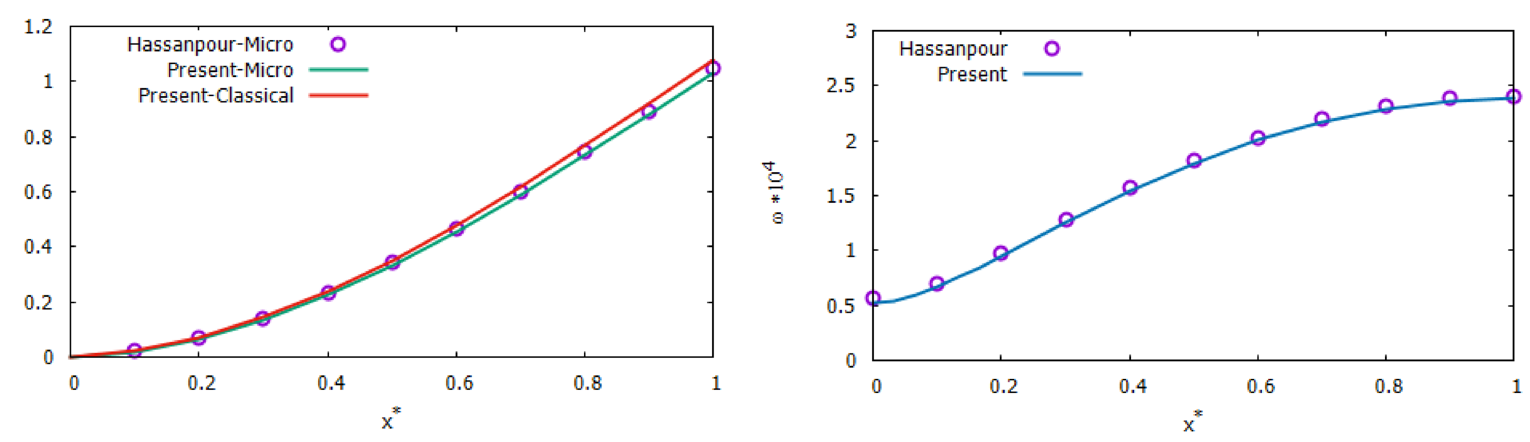
$$\varepsilon = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{21} & \varepsilon_{31} \\ \varepsilon_{12} & \varepsilon_{22} & \varepsilon_{32} \\ \varepsilon_{13} & \varepsilon_{23} & \varepsilon_{33} \end{bmatrix} \quad \chi = \begin{bmatrix} \chi_{11} & \chi_{21} & \chi_{31} \\ \chi_{12} & \chi_{22} & \chi_{32} \\ \chi_{13} & \chi_{23} & \chi_{33} \end{bmatrix} \quad \begin{aligned} \varepsilon_{ij} &= u_{i,j} + e_{ijk}\omega_k \qquad i,j,k = 1{,}2{,}3 \\ \chi_{ij} &= \omega_{i,j} \end{aligned}$$

Results



$$\frac{\alpha}{E} = 10^{-2} \qquad \frac{\gamma}{E} = \frac{\varepsilon}{E} = 2.5 * 10^{-4}$$

$$\frac{P}{EA} = 5 * 10^{-6} \qquad \frac{L}{\sqrt{\frac{I}{A}}} = 10$$



Publications

- A. Pagani, E. Carrera, R. Augello (2018), "Evaluation of geometrically nonlinear effects due to large cross-sectional deformations of beam-like structures", Submitted.
- R. Augello, E. Carrera, A. Pagani (2018), "Investigation about consistency of different nonlinear geometric relations in the analysis of beams and thin-walled structures", Submitted.
- A. Pagani, E. Carrera, R. Augello (2018), "Frequency and mode change in the large deflection and post-buckling of compact and thin walled beams", *Journal of Sound and Vibration* 432 (2018): 88-104.
- A. Stio, P. Spinolo, E. Carrera, R. Augello (2016), "Analysis of landing mission phases for robotic exploration on Phobos Mars's Moon", Advances in Aircraft and Spacecraft Science, Vol. 4 no. 5, pp. 529-541.

Objectives

- Formulation of advanced theories of structures and elasticity models based on CUF and nonloncal theories (perydinamics or couple-stress theories);
- Apply the developed models to practical engineering problems (smart structures and biostructures);
- Adding geometrical nonlinear analysis on these models to take into account any problems in the large displacement field.
- Collaboration with City University of Hong Kong.

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