# Modelling and design of multi-stable composite structures

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## Topic

The goal of this research project is to design continuum structures capable of morphing between different stable shapes without the need for complex mechanisms or conventional actuators. Morphing components are often initially-curved nonlinear structures undergoing large displacements and rotations in a multi-physics (thermal or piezoelectric) environment. The mechanical behaviour of such structures is investigated via advanced one-dimensional finite elements based on Carrera Unified Formulation (CUF) accounting for initial curvatures and geometric nonlinearities.

## Motivation

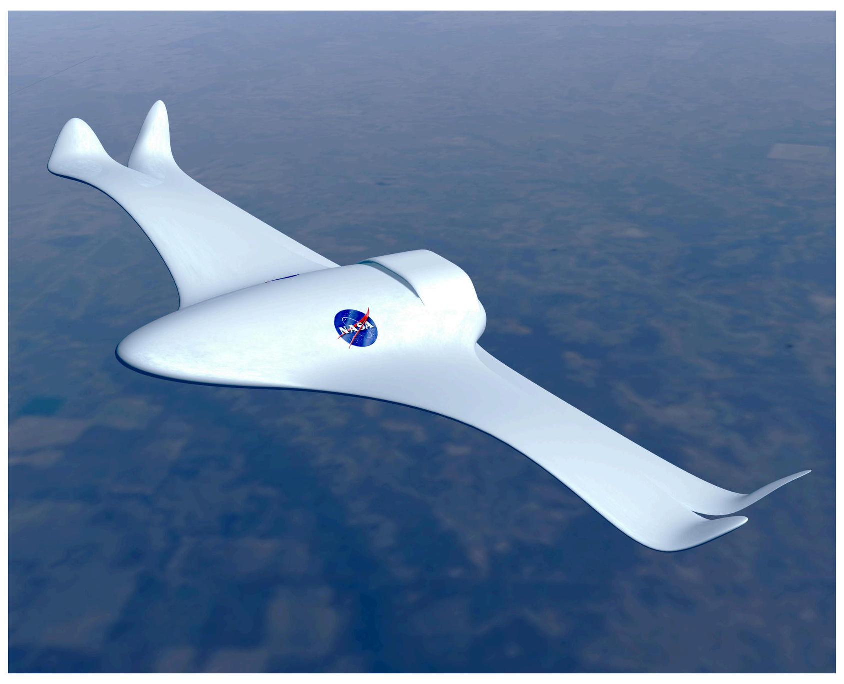

**Figure 1:** NASA morphing aircraft concept

Multistable structures may represent an effective solution for future morphing engineering designs accommodating a wide range of service conditions. Potential applications encompass the field of deployable architectural structures, aeronautics (shape-changing aerodynamic panels, variable geometry engine exhausts and reconfigurable airplane wings), optics (shape-changing mirrors for active focusing in adaptive optical systems), and micro electromechanical systems (micro-switches, mechanical memory cells, valves, micro-pumps). Due to the complex geometries and the highly non-linear behaviours, at present, models predicting multistability are often characterised by a strong compromise between computational costs and results accuracy. CUF-based one-dimensional finite elements can represent a reliable tool for a computationally efficient 3D-like mechanical response prediction of morphing composite structures.

## Main Objectives

- Assessment of CUF-based one-dimensional finite elements for the 3D multi-physics analysis of composite structures
- Development of advanced structural models for the analysis of initially-curved composite structures
- Development of CUF-based geometrically nonlinear finite elements for a 3D-like accuracy in large displacements and buckling analyses
- Investigation of the snap-through behaviour of bi-stable composites
- Design of multi-stable composite structures aimed at industrial exploitation

## Methodology

- Principle of Virtual Displacements

$$\delta\mathscr{L} = \delta\mathscr{L}_{int} - \delta\mathscr{L}_{ext} = 0$$

$$\delta\mathscr{L}_{int} = \int_{V_0} \delta\mathbf{E}^T\mathbf{S}dV$$

- Carrera Unified Formulation

$$\mathbf{u}\left(x,y,z\right) = F_\tau\left(y,z\right)\mathbf{u}_\tau\left(x\right) \quad \text{with} \quad \tau = 1,\ 2,\ \ldots,\ N_u$$

- Fundamental nucleus for 1D CUF-based curved finite elements

$$\begin{aligned} K_{ss}^{\tau sij} = \; & \kappa^2 C_{55}\, J_{\tau t}\, I_{ij}\,/H + \kappa\, C_{56}\, J_{\tau t,_\eta}\, I_{ij} + \kappa\, C_{55}\, J_{\tau t,_\xi}\, I_{ij} + \kappa\, C_{66}\, J_{\tau,_\eta t}\, I_{ij} + \\ & H\, C_{66}\, J_{\tau,_\eta t,_\eta}\, I_{ij} + H\, C_{66}\, J_{\tau,_\eta t,_\xi}\, I_{ij} + \kappa\, C_{55}\, J_{\tau,_\xi t}\, I_{ij} + H\, C_{56}\, J_{\tau,_\xi t,_\eta}\, I_{ij} + \\ & H\, C_{55}\, J_{\tau,_\xi t,_\xi}\, I_{ij} + \kappa\, C_{15}\, J_{\tau t}\, I_{i,sj}\,/H + C_{16}\, J_{\tau t,_\eta}\, I_{i,sj} + C_{15}\, J_{\tau t,_\xi}\, I_{i,sj} + \\ & \kappa\, C_{15}\, J_{\tau t}\, I_{ij,s}\,/H + C_{16}\, J_{\tau,_\eta t}\, I_{ij,s} + C_{15}\, J_{\tau,_\xi t}\, I_{ij,s} + C_{11}\, J_{\tau t}\, I_{i,sj,s}\,/H \end{aligned}$$

- Geometrically non-linear advanced finite elements

$$\begin{aligned} d\left(\delta\mathscr{L}\right) = \; & \delta\mathbf{q}_{\tau i}^T \int_{V_0^e} \mathbf{G}_{\tau i}^T \left\{\left[\mathbf{H}^T + \mathbf{A}^T\left(\boldsymbol{\theta}\right)\right]\mathbf{Q}\left[\mathbf{H} + \mathbf{A}\left(\boldsymbol{\theta}\right)\right] + \hat{\mathbf{S}}\right\}\mathbf{G}_{\sigma j} dV\, d\mathbf{q}_{\sigma j} = \\ & \delta\mathbf{q}_{\tau i}^T\left(\mathbf{K}_{\tau\sigma ij}^{el} + \mathbf{K}_{\tau\sigma ij}^{et1} + \mathbf{K}_{\tau\sigma ij}^{et2}\right) d\mathbf{q}_{\sigma j} \end{aligned}$$

## Results

### Solid-like prediction of thermal stresses in composite structures via 1D CUF [1]

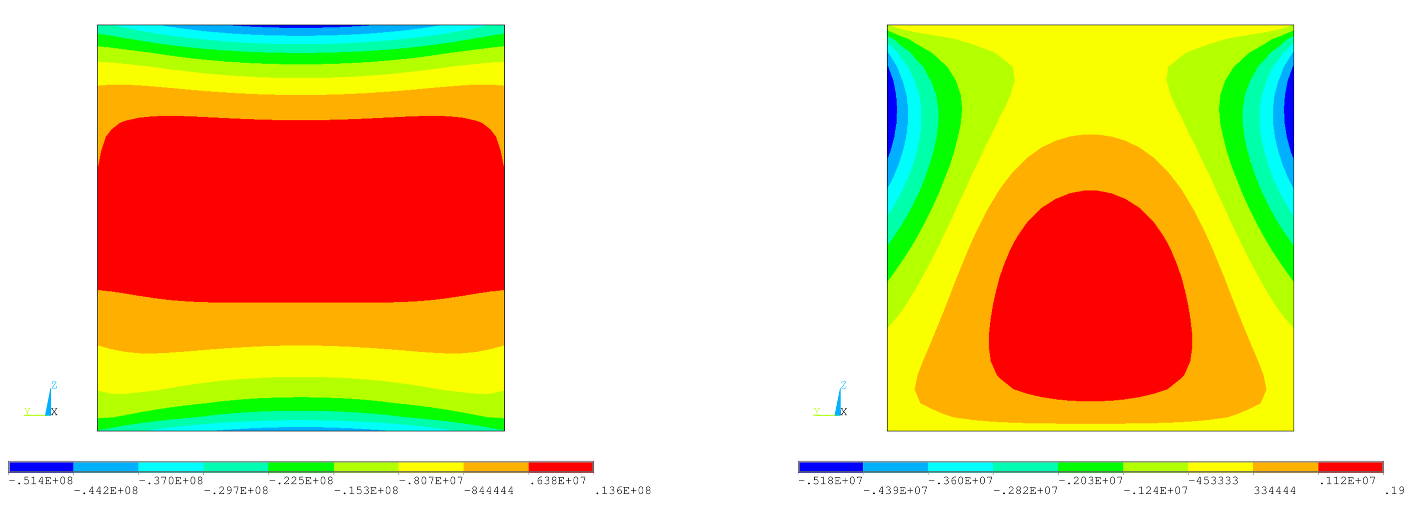

**Figure 2:** Axial and shear stresses in Functionally Graded cantilever beams via 1D N=13 CUF model (validated towards 3D FEM)

### Accurate displacement and stress predictions in geometrically nonlinear analyses via higher-order theories [2]

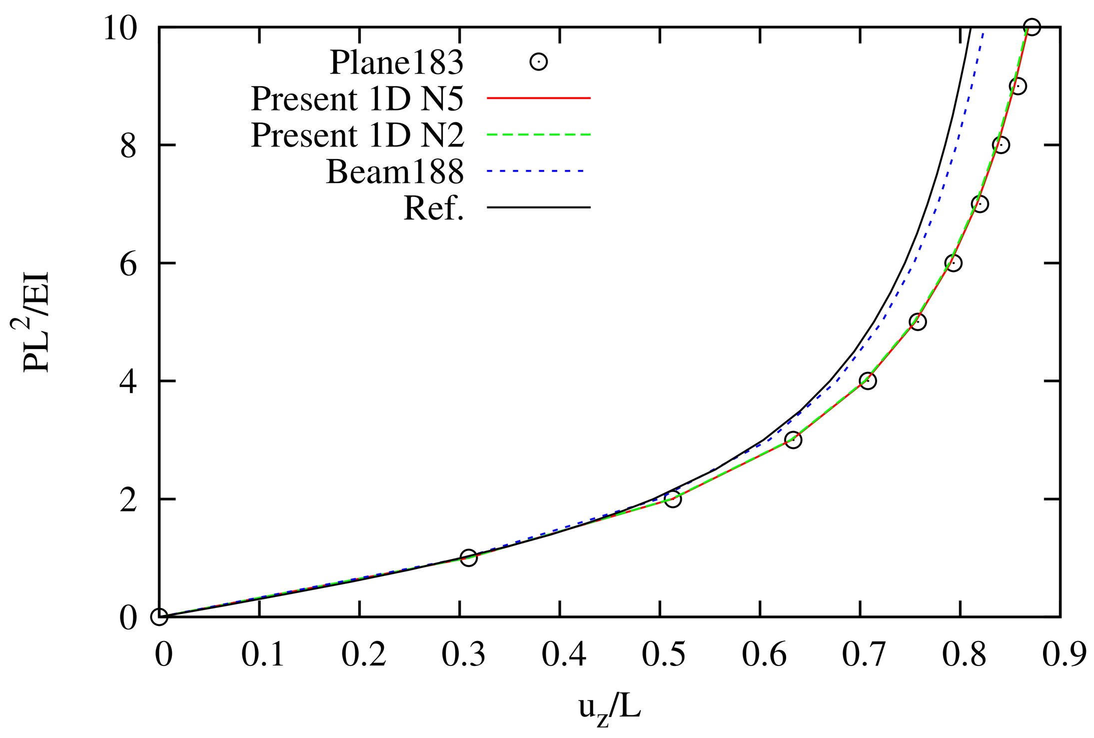


**Figure 3:** Load vs. transverse displacement for a thick cantilever beam

| | $\tilde{\sigma}_{xx}$ | | $\tilde{\sigma}_{xz}$ | | $DOFs$ |
|---|---|---|---|---|---|
| | $p_0$ = 3 MPa | $p_0$ = 9 MPa | $p_0$ = 3 MPa | $p_0$ = 9 MPa | |
| ABAQUS 3D | 959.42 | 1580.03 | 40.99 | 80.02 | $5.7 \cdot 10^5$ |
| Present 1D N8 | 1000.79 | 1561.06 | 40.85 | 75.88 | $1.1 \cdot 10^3$ |
| Present 1D N5 | 1001.19 | 1562.35 | 39.15 | 72.80 | $4.8 \cdot 10^2$ |
| Present 1D N2 | 1010.19 | 1587.51 | 27.21 | 51.22 | $3.7 \cdot 10^2$ |

**Table 1:** Axial and shear stress prediction in an asymmetric laminated cantilever beam via 1D CUF finite elements

### CUF-based 1D locking-free curved finite elements

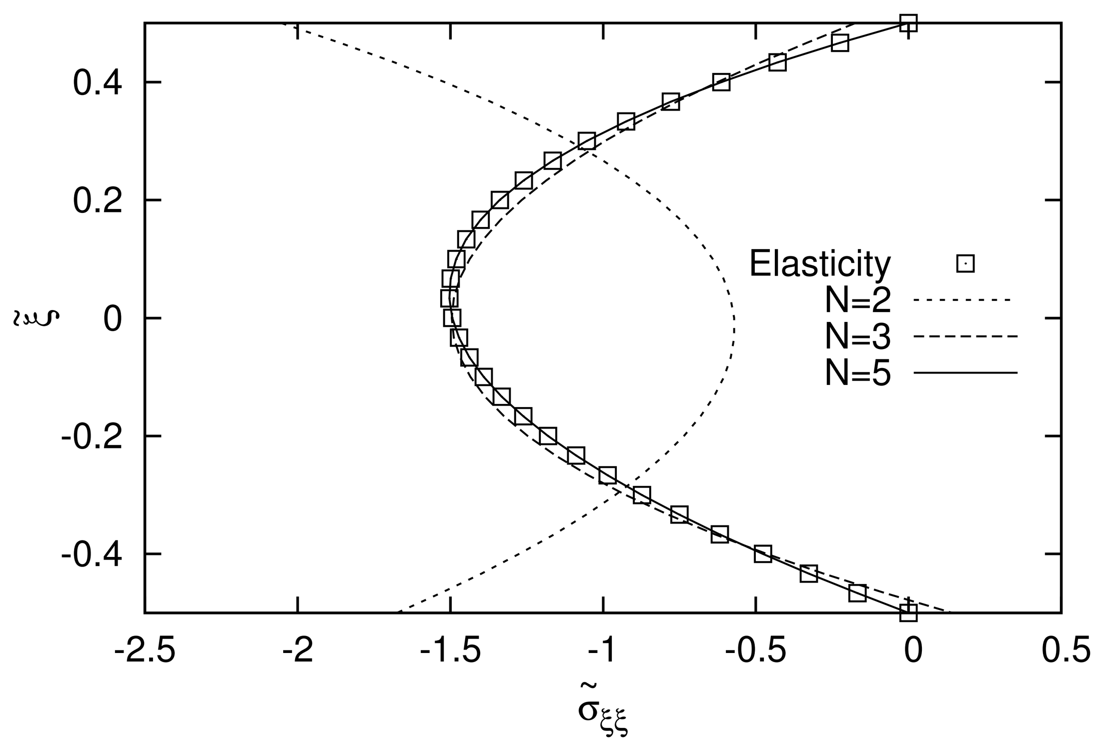


**Figure 4:** Through-the-thickness radial stress in a thick cantilever circular beam

## Forthcoming Research

Design of multi-stable composite components will be carried out by use of the proposed formulation and possible concepts for applications in morphing structures will be investigated.

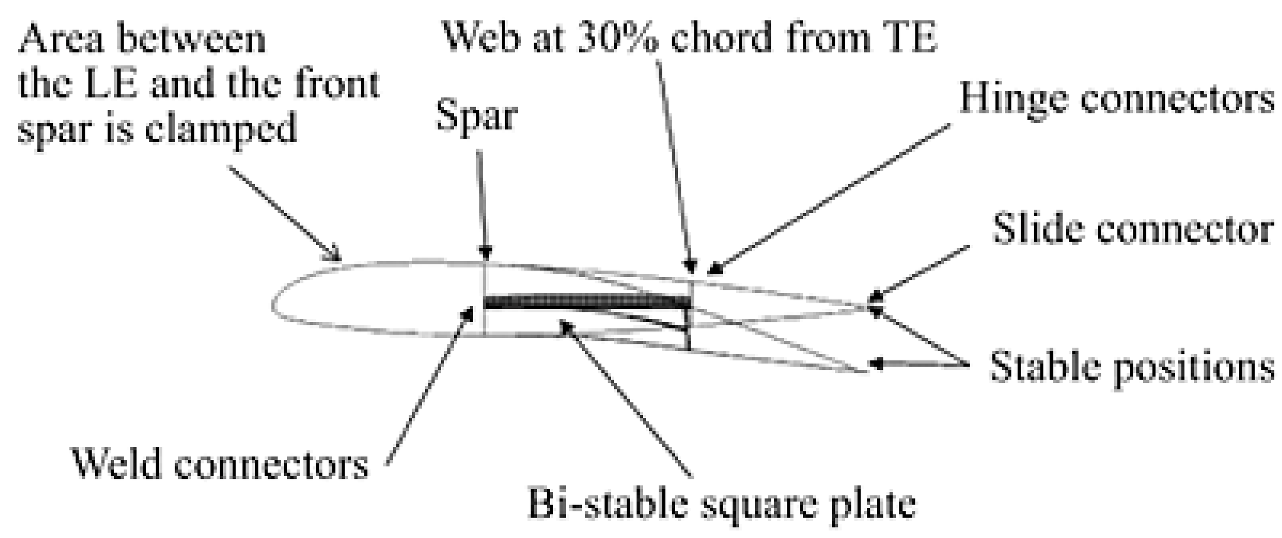


**Figure 5:** Morphing trailing edge using bistable composite plate. (Lachenal et al.)

## Acknowledgements

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## References

[1] G. De Pietro, Y. Hui, G. Giunta, S. Belouettar, E. Carrera, and H. Hu. Hierarchical one-dimensional finite elements for the thermal stress analysis of three-dimensional functionally graded beams. *Composite Structures*, 153:514–528, 2016.

[2] G. De Pietro, G. Giunta, S. Belouettar, A. Pagani, and E. Carrera. Geometrically nonlinear hierarchical finite elements via a unified formulation. In *Proc. 20th International Conference on Composite Structures (ICCS, Paris, France, September 2017)*, 2017.